Crash Risk and Risk Neutral Densities

Ren-Raw Chen
Gabelli School of Business
Fordham University
45 Columbus Avenue
New York, NY 10019
rchen@fordham.edu

Pei-lin Hsieh
Wang Yanan Institute for Studies in Economics (WISE)
Xieman University
D316, Economics Building, Xiamen University
Xiamen, 361005 China
peilin.h@gmail.com

Jeffrey Huang
Global Markets
KGI Bank
125, Nanking East Road, Section 5
Taipei, Taiwan
Jeffreycc.Huang@kgi.com

June 1, 2017

Abstract

Recent asset pricing literature has focused on "crash risk". In particular, Gabaix et. al. (2008) and Farhi and Gabaix (2016) study the crash risk in the currency market. Gabaix et. al. (2016) uncover that since the Fall of 2008, "crash risk" has increased dramatically, implied by the FX options data. Motivated by Gabaix et. al. and the literature, and furthermore the recent troubles in the Euro zone (since 2008), we use the EUR/USD exchange rate to study the information contents of its RNDs since the crisis. We study the EUR/USD exchange rate risk-neutral density (RND) that results in a number of novel findings.

Using daily data from EUR/USD FX options during the period from January 2, 2008 till March 18, 2015, we discover that RND (especially higher moments) has superior explanatory powers in predicting and explaining crash risk and its risk premiums. Furthermore, our empirical results show that the higher moments of RND co-move closely with macroeconomic variables. In all cases, we find moments outperform the implied volatility from the Black-Scholes model.

In addition, we also estimate the elasticity parameter β assuming a CEV process for the variance and the result indicates that β is quite high (1.88) which is substantially more than the Heston model (where β is 1). We find that the term structure of volatilities derived from the RNDs rejects the Heston model and any Gaussian-based models (such as the Black-Scholes) for the EUR/USD FX rate.

Key words: European crisis, subprime crisis, crisis risk, risk neutral density, FX option, stochastic volatility, Heston model

JEL Classification: G12, G14

1 Introduction

The information contents of implied volatility pioneered by Chiras and Manaster (1978) and Canina and Figlewski (1993) has sparked a wide interest in learning how option-implied parameters, which are forward-looking by definition, can carry useful information of the future of the financial markets. Instead of retrieving one single parameter (implied volatility), recent studies retrieve the entire risk-neutral density (RND) function. There are two advantages of using RND. First, RND is model-free (in loose terms) as opposed to the implied volatility that requires a parametric model (such as Black-Scholes). Secondly, the multiple moments of the RND carry much more granular information than a single volatility number. In particular, the skewness that is highly correlated with risk premiums and kurtosis that reflects fat tails provide investors much more useful information than the volatility.

Given that RND carries useful forward-looking information, the literature has in general used RND for in the following areas. The first strand of literature is use RND for prediction. This includes the prediction of (1) future movements of the underlying asset (e.g. Gemmill and Saflekos (2000)); (2) future option prices (e.g. Khrapov (2014)); (3) future volatility/variance (e.g. Jiang and Tian (2005));[1] and (4) future distributions of the underlying asset (e.g. Xu and Taylor (1994) and Chen and Gwati (2012) for the volatility term structure and Christoffersen and Mazzotta (2005) for the entire density.

The second strand is reflecting economic events. This includes Datta, Londono, and Ross (2016) who study RNDs estimated around episodes of high geopolitical tensions, oil supply disruptions, and macroeconomic data releases in the oil market; Malz (1997) who studies the peso problem; Cooper and Talbot (1999) who study the yen crisis in the September of 1998; Birru and Figlewski (2012) and Chen and Gwati (2012) who study the global meltdown of 2008; Melick and Thomas (1997) who study the Gulf crisis in 1990; Gemmill and Saflekos (2000) who study major economic events such as 1987 crash and British elections; Castren (2004) who finds interventions on the exchange rate coincide with systematic changes in all moments of the estimated RNDs and finds that RND moments from three newly joined E.U. member states (Poland, Czech Republic and Hungary) move around policy news (Castren (2005)); and lastly Kitsul, Yuriy, and Wright (2013) who find that RNDs assign considerably more mass to extreme outcomes (either deflation or high inflation) than do their time series counterparts.

[1] Also see Christensen and Prabhala (1998) and Jorion (1995).

The third stand is explaining risk premiums. This is represented by Dennis and Mayhew (2002) who discover that the skewness of the RND tends to be more negative for stocks that have larger betas, suggesting that options contain useful information about the market risk. Bliss and Panigirtzoglou (2002) directly estimate risk aversion using the RND by option prices. Malz (1997) finds higher moments explain currency excess returns much better than the CAPM. Similarly, Chen and Gwati (2013) also find that higher moments consistently explain subsequent currency excess returns for horizons between one week to twelve months. Nitteberg (2011) finds RNDs to be highly skewed and claim evidence of market sentiment in FX and Oil markets respectively. Breuer (2011) argues that the volatility of implied volatility are considered here as a proxy for the risk premium and studies how the volatility risk premium affect the informational content of currency options. Jurek and Wu (2014) study currency risk premiums by comparing RNDs with parametrically estimated currency dynamics and conclude that option-implied currency risk premiums provide an unbiased forecast of monthly currency excess returns. Similarly, Ait-Sahalia, Wang, and Yared (2001) who compare RNDs with historically estimated density functions and reject the hypothesis that the S&P 500 options are efficiently priced given the S&P 500 index dynamics.[2] Carr and Wu (2007) and Bakshi, Carr, and Wu (2008) find strong skewness in RNDs over time derive models to accommodate stochastic skewness and risk premiums.

Gabaix et. al. (2016) uncover that since the Fall of 2008, "crash risk" has increased dramatically, implied by the FX options data. Motivated by Gabaix et. al. and the literature, and furthermore the recent troubles in the Euro zone (since 2008), we use the EUR/USD exchange rate to study the information contents of its RNDs since the crisis.[3] We believe that the option implied RND can provide good insights, or even early warnings of the turbulences. In this paper, we study information contents of the risk-neutral densities (RND) implied by EUR-USD foreign currency (FX) option prices. The data cover the period from January 2, 2008 till March 18, 2015. For each day, prices of 40 options (5 moneyness levels and 8 maturities) are reported on Bloomberg. We discover the following: (1) the third and the fourth moments of the RND have a substantial explanatory power of FX swap spreads that represent term risk premiums, relative to the implied volatility; (2) the third and fourth moments of the RND can successfully predict catastrophic events (Lehman crisis, Flash Crash, and European sovereign crisis);[4] (3) the third and the fourth moments of the RND can predict future FX rates well; (4) shorter term fourth

[2]Note that Ait-Sahalia and Lo (1998) have estimated density function using a non-parametric method. This is different from using option prices and a separate line of literature.

[3]Farhi and Gabaix (2016) relate rare disasters and exchange rate moves.

[4]In a broader measure, we use the VIX index to proxy rare events.

moments (less than 3 months) are driven by speculative activities and yet longer term fourth moments (greater than 3 months) are driven by imports and exports; (5) higher order moments can predict the Economic Policy Uncertainty index and the USD influence index; and (6) fourth moments outperform third and second moments in predicting realized volatility.

In addition, we compare the volatility term structure implied by the RND with popular parametric models such as Black-Scholes and Heston. In particular, we find that both Black-Scholes and Heston models fail to explain the volatility term structure by a wide margin. This is similar to the idea of Britten-Jones and Neuberger (2000). We also estimate the elasticity parameter to the variance process and the result indicates that the elasticity is much higher than the Heston model.

Retrieving risk-neutral densities (RND) from option prices is a challenging exercise and has attracted a lot of attention, in that the number of options traded in the market place is finite and hence the RND is not uniquely defined. In the large body of literature, Britten-Jones and Neuberger (2000) and Jackwerth and Rubinstein (1996) are most related to our paper, yet our proposed RND method is simpler and faster in estimating model-free moments. Our next section shall briefly illustrate other popular methods and discuss the difference between our approach and the approaches of Britten-Jones and Neuberger (2000) and Jackwerth and Rubinstein (1996). Though seeking a methodology that can best retrieve information from option prices could be an interesting research topic on its own right, it is not the main focus of our paper. We have implemented the major popular methods in the literature for our empirical study as a robust check.[5]

There are literally an infinite number of ways to retrieve the RND (to be discussed in details in the next section). In this paper, we adopt a piece-wise flat RND that matches exactly the options traded in the marketplace. There are two advantages of using the piece-wise flat RND. First, it is shown empirically, when compared to more complicated methods, such RNDs are very stable over time. Second, the RND is a closed-form solution which is easy and fast to solve. Third, the number of options can vary from one maturity to another (a term structure of RNDs) and yet by construction all options are priced perfectly.

[5] The results are available upon request and we have a short discussion of these results.

2 Methodology

The pioneer work of Breeden and Litzenberger (1978) demonstrates that the probability distribution of the price/return of an asset can be derived from its options (e.g. calls) by taking the derivative of the option price with respect to the strike price as follows:

$$F(S) = \frac{\partial C}{\partial K} \tag{1}$$

where C is the option (e.g. call), K is the strike, and $F(S)$ is the cumulative density function (c.d.f.) for the underlying asset S. In the case of the Black-Scholes model, equation (1) equals $N(d_2)$.[6] While a c.d.f. is all we need to price an option, it is more intuitive to write a p.d.f. (probability density function):

$$f(S) = \frac{\partial F(S)}{\partial K} \tag{2}$$

Bakshi and Madan (2000), among others, then show that in such a case a continuum of option prices (across both maturities and strikes) can therefore completely span the asset state space. That is, these European options can duplicate any exotic options on the same underlying asset. In reality, of course, we do not have an infinite number of European options but only a small set of such contracts. As a result, various methods are proposed to remedy the lack of enough option prices. The literature has used four different methods:

1. smoothing in the volatility space
2. smoothing in the price space
3. smoothing using a parametric model
4. smoothing in the density space

Volatility smoothing has been the most popular method used in identifying the RND. Fig-

[6] We assume that readers have good familiarity of the Black-Scholes model where $N(d_2)$ is the in-the-money probability of the European call option. In fact, within the option community, $N(d_1)$ (which is know as delta) and $N(d_2)$ are frequently used expressions.

lewski (2002) provides an excellent review of the literature.[7] Under this approach, due to the forced smoothness of the fitted volatility function, not all options can be exactly repriced.[8] Nevertheless, by enforcing a smooth function through the volatilities, a number of “fake options” are created in order to fulfill equation (1).

In contrast, few researchers use price smoothing. Interested readers can find excellent references in Figlewski (2008) and Orosi (2015).

Many researchers have tried to fit a parametric model to data. The problem with this approach is of course that not all options are priced correctly. Prices observed in the market are assumed to have measurement errors; or equivalently, they are not true prices (i.e. model prices). The benefits of using the parameter models, as argued in the literature, are (1) parameters have meanings (e.g. mean reversion, correlation, volatility), and (2) consistency over time is imposed. The drawback of the approach is apparently that the model price is assumed to provide the true price and market prices are assumed to be incorrect. While this is a more desirable approach in terms of option repricing, the degrees of freedom are not easily managed. Either there will be over or under identification of the RND generated by the binomial model (or any lattice). Furthermore, as one shrinks the time interval of the binomial model, the number of degrees of freedom increases, worsening the identification problem.[9][10][11]

Assuming all market prices are correct, researchers retrieve the RND directly from option prices. Note that, due to the insufficient number of option prices and too much flexibility in the functional form of the RND, there exist more than one RND that can price existing options (i.e. RND is not unique, unless there is a continuum of options). In this paper, we adopt a piece-wise constant RND function. Piece-wise constant functions are commonly found in industry.

[7]For example, Shimko (1993) fits the volatility smile with polynomials; Malz (1997); Campa, Chang, and Reider (1997) adopts a cubic spline function for the implied volatility curve; Bliss and Panigirtzoglou (2002) use a weighted natural spline to fit an implied volatility curve; Jondeau and Rockinger (2000) experiment various methods including Hermite polynomials, Edgeworth expansion. Santos and Guerra (2015) summarize

[8]Usually liquid options (e.g. those that are near the money) are given heavier weights and illiquid options (e.g. farther away from at-the-money) are given lighter weights.

[9]The popular choices for the model are the binomial model (implied binomial tree) by Rubinstein (1994) and Campa, Chang, and Reider (1997); trinomial model Derman and Kani (1994); mixture of normals by Melick and Thomas (1997), Campa, Chang, and Reider (1997), Cooper (1999) and Gemmill and Saflekos (2000). Derman and Kani (1994)) further impose smoothness on the lattice, which makes their method indistinguishable from volatility smoothing.

[10]To fit a stochastic process, see Jondeau and Rockinger (2000) who try jump-diffusion and Heston's stochastic volatility model.

[11]Some may argue that perfect fit is not necessary as bid-offer spreads are sometimes wide. While this argument is valid, as a modeler, we would like our models to be able to fit any value precisely.

It has the advantage of stability and ease (fast) to compute. More importantly, in our case, it offers an exact solution to the RND given any number of options available in the marketplace. Furthermore, as it turns out, that piece-wise constant RNDs preserve the most information from option prices. Other higher power polynomials (we also use piece-wise linear and cubic-spline as a robust check) overfit the density and result in losing useful information.

An illustration of the density function is given in Figure 1.[12] In Figure 1, the density function is plotted upside down in order to match with the option payoffs. For each option C_k (as defined by the strike K_k), there is a corresponding density mass a_k which is a constant. Note that $0 \leq k \leq n+1$ where n is the number of options; and $K_0 = 0$ and $K_{n+1} = x$ which are the lower and upper limits of the RND.

[Figure 1 Here]

Formally we write the RND as (where a subscript is added to the density function to reflect that the RND is piece-wise):

$$f_k(S) = a_k \quad K_k < S < K_{k+1} \tag{3}$$

for $0 \leq k \leq n$ and $K_{n+1} = x$. This RND is then calibrated to option prices in the market. The call option pricing equation can be easily derived as follows:

$$\begin{aligned} C_k &= \int_{K_k}^{x} (S - K_k) f_k(S) dS \\ &= \frac{1}{2} \sum\nolimits_{i=k}^{n} a_i (K_{i+1}^2 - K_i^2) - K_k \sum\nolimits_{i=k}^{n} a_i (K_{i+1} - K_i) \end{aligned} \tag{4}$$

As a result, by equation (1), we have:

$$\frac{C_k - C_{k-1}}{K_k - K_{k-1}} = \frac{1}{2} a_{k-1}(K_k + K_{k-1}) - \left\{ a_{k-1} K_{k-1} + \sum\nolimits_{i=k}^{n} a_i (K_{i+1} - K_i) \right\} \tag{5}$$

[12] A piece-wise linear, log-linear, and cubic spline examples are given in the Appendix where a comparison of the RNDs and their variances are provided.

which, if used recursively, yields the following solution for the probability mass:

$$a_k = \frac{2}{(K_{k+1} - K_k)^2} \left[C_k - \frac{1}{2} \sum\nolimits_{i=k+1}^{n} a_i \left[(K_{i+1}^2 - K_i^2) - 2K_k(K_{i+1} - K_i) \right] \right] \tag{6}$$

with $a_n = 2C_n/(x - K_n)^2$.

Since the probabilities must sum to 1, i.e. $\sum_{j=0}^{n} a_j \left[K_{j+1} - K_{j+1} \right] = 1$, the upper limit of the RND can be solved as:

$$x = K_n + \frac{1}{a_n} \left[1 - \sum\nolimits_{i=1}^{n} a_{i-1}(K_i - K_{i-1}) \right] \tag{7}$$

where $K_0 = 0$. Note that equations (21) and (6) must be solved iteratively to reach a convergence.

Having the RND estimated (i.e. a_k for $k = 0, \cdots, n$), we can then study a number of distributional issues related to the RND. Firstly, we can calculate all the moments of the RND for any given maturity (and we have 8 maturities to form a term structure of RNDs). Then, we study the behavior of the stochastic volatility process. As a time series of RNDs under a given maturity subsumes a random process of the volatility as indicated by Britten-Jones and Neuberger (2000), it would be interesting to know how this volatility process behaves. For example, it is interesting to know if the implied volatility process is consistent with the Heston model.

3 Empirical Results

3.1 Data

Our data contain EUR/USD options from January 2, 2008 till March 18, 2015 (1,847 days). On each day, there are 8 maturities (1-week, 2-week, 1-month, 2-month, 3-month, 6-month, 9-month, and 1-year) and 5 moneyness levels (10-delta, 25-delta, 50-delta (ATM), 75-delta, and 90-delta) a total of 40 options, quoted by Bloomberg.

Different from equity options, FX options are quoted by the following conventions:

- by deltas and not by strikes, like 10-delta and 25-delta
- by a strategy and not by a naked option, like risk-reversal (RR) and butterfly (BF)
- by (Black) volatility and not by premium (similar to interest rate options)
- by rolling maturities as opposed for fixed maturities[13]

As a result, we must back out the option premiums by strike. Such a conversion is standard. The data we obtained have already been converted by the vending bank (KGI bank).[14]

In addition to the option data, we also collect the spot FX rates and FX swap rates. The FX swap rates are quoted as spreads to the current spot. The term structure contains: overnight (ON), tomorrow-next (TN), spot-next (SN), 1-week (1W), 2-week (2W), 3-week (3W), 1-month (1M), 2-month (2M), 3-month (3M), 4-month (4M), 5-month (5M), 6-month (6M), 9-month (9M), 1-year (1Y), and 2-year (2Y). These terms to maturity vest the maturities of the options and hence, we can only use those that match with option maturity tenors.

Given that both option premiums and strikes need to be computed from market quotes which are by delta and Black's implied volatility, the strike price changes from contract to contract, even though they have the same Black's delta. For example, a 25D (25-delta or delta=0.25), 3-month option on January 2, 2008 is quoted as 9.1125% (Black's implied volatility). Given the spot price to be 1.4715, the price and the strike are computed to be 0.009795 and 1.518760 respectively. On the next day, the spot price is 1.4750, the same 3-month, 25D call option is quoted as 9.3000%, which give the option premium and strike price as 0.010013 and 1.523382 respectively. As a result, as the market moves, strike price also moves in order for the option to be 25D (delta=0.25).

Since option price, underlying spot price, and strike price all move as the market moves, it is best to summarize the data using moneyness and percentage premium (i.e. scale the strike price and option premium by the spot price). In Table 1, we summarize for each of the eight maturities (Panels 1W $\sim$ 1Y) basic statistics of moneyness (i.e. strike/spot) and option premium as a percentage of spot (i.e. option/spot). We note that moneyness is near 1 at 50D (50-delta) for all maturities, as it should.[15] As maturity increases, moneyness decreases for in-the-money

[13] That is, unlike equity options that expire on a Saturday after the third Friday in the expiry month, FX options always expire a given time to maturity from the trade day.

[14] We are grateful to the KGI bank for their generosity.

[15] Both mean and median are 1 with almost no standard deviation.

options and increases for out-of-the-money options (for example, the average moneyness for 90D is 0.979 for 1W and gradually decreases to 0.838 for 1Y; and for 10D is 1.020 for 1W and 1.171.) This is not surprising, because as maturity lengthens, the uncertainty of the underlying spot increases, and in order for the option to be 90D, strike price has to be lower.[16] Same analysis is applied to low deltas.

[Table 1 Here]

3.2 Risk-Neutral Density (RND)

First of all, we examine the behaviors of the risk-neutral densities (RND). These RNDs are retrieved from FX options directly (using either equation (4) or (5)). On each day there are 8 RNDs, one for each maturity. Then we can derive the moments as follows:

$$
\begin{aligned}
\mathbb{E}[S^i] &= \int_0^x S^i f(S) dS \\
&= \sum\nolimits_{j=0}^{n} \int_{K_j}^{K_{j+1}} S_j^i f(S_j) dS_j \\
&= \sum\nolimits_{j=0}^{n} a_j \int_{K_j}^{K_{j+1}} S_j^i dS_j \\
&= \sum\nolimits_{j=0}^{n} a_j \frac{1}{i+1} \, S_j^{i+1} \Big|_{K_j}^{K_{j+1}} \\
&= \frac{1}{i+1} \sum\nolimits_{j=0}^{n} a_j \left(K_{j+1}^{i+1} - K_j^{i+1} \right)
\end{aligned}
\tag{8}
$$

There have been a series of studies that focus on retrieving skewness from RND, such as Carr and Wu (2007) and Bakshi, Kapadia, and Madan (2000). Our computation is far more parsimonious as our RND is piece-wise flat.

It is also convenient to derive the log moments. Let $X = \ln S$. Then,

[16] One can gain good intuition from the Black-Scholes model. Option price as a function of the underlying asset is a monotonically increasing convex function. The longer is the maturity, the farther left is the function, which implies a smaller strike price for the same delta.

Table 1: Summary Statistics for FX Options' Delta and Premium

	K/S					C/S				
	90D	**75D**	**50D**	**25D**	**10D**	**90D**	**75D**	**50D**	**25D**	**10D**
1-Week										
mean	0.979	0.990	1.000	1.010	1.020	0.022	0.013	0.006	0.002	0.001
median	0.980	0.990	1.000	1.009	1.018	0.020	0.012	0.006	0.002	0.001
std.dev.	0.008	0.004	0.000	0.004	0.008	0.008	0.005	0.002	0.001	0.000
skew	-1.443	-1.428	1.495	1.582	1.648	1.440	1.436	1.457	1.507	1.574
kurt	3.195	3.236	14.837	3.674	3.770	3.164	3.237	3.295	3.384	3.474
2-Weeks										
mean	0.970	0.985	1.000	1.014	1.028	0.031	0.018	0.008	0.003	0.001
median	0.972	0.986	1.000	1.013	1.026	0.029	0.017	0.008	0.003	0.001
std.dev.	0.011	0.005	0.000	0.006	0.011	0.011	0.006	0.003	0.001	0.000
skew	-1.301	-1.301	0.226	1.566	1.684	1.297	1.312	1.366	1.469	1.585
kurt	2.560	2.700	6.204	3.540	3.844	2.521	2.703	2.923	3.186	3.460
1-Month										
mean	0.955	0.978	1.000	1.021	1.042	0.046	0.027	0.012	0.005	0.001
median	0.957	0.979	1.000	1.020	1.039	0.044	0.026	0.012	0.004	0.001
std.dev.	0.016	0.007	0.001	0.008	0.017	0.016	0.009	0.004	0.002	0.001
skew	-1.056	-1.063	-0.119	1.512	1.718	1.054	1.083	1.190	1.375	1.568
kurt	1.530	1.738	5.039	3.231	3.967	1.470	1.712	2.203	2.815	3.407
2-Months										
mean	0.936	0.969	1.001	1.031	1.060	0.066	0.038	0.017	0.006	0.002
median	0.938	0.969	1.001	1.029	1.056	0.065	0.037	0.017	0.006	0.002
std.dev.	0.021	0.010	0.001	0.011	0.023	0.021	0.012	0.006	0.002	0.001
skew	-0.760	-0.751	-0.803	1.328	1.565	0.767	0.801	0.929	1.151	1.362
kurt	0.576	0.761	2.163	2.398	3.239	0.511	0.745	1.284	1.973	2.572

$$
\begin{aligned}
\mathbb{E}[\ln S^i] = \mathbb{E}[X^i] &= \int_{-\infty}^{x} X^i g(X) dX = \int_{-\infty}^{x} X^i \left\{ f(S) \tfrac{dS}{dX} \right\} dX \\
&= \sum\nolimits_{j=0}^{n} a_j \int_{\ln K_j}^{\ln K_{j+1}} X^i e^X dX \\
&= \sum\nolimits_{j=0}^{n} a_j \left[X^i e^X \Big|_{\ln K_j}^{\ln K_{j+1}} - \int_{\ln K_j}^{\ln K_{j+1}} i X^{i-1} e^X dX \right] \qquad (9) \\
&= \sum\nolimits_{j=0}^{n} a_j \left[K_{j+1} \left(\ln K_{j+1}\right)^i - K_j \left(\ln K_j\right)^i \right] - i \sum\nolimits_{j=0}^{n} a_j \int_{\ln K_j}^{\ln K_{j+1}} X^{i-1} e^X dX
\end{aligned}
$$

Table 1 (continued): Summary Statistics for FX Options' Delta and Premium

	K/S					C/S				
	90D	**75D**	**50D**	**25D**	**10D**	**90D**	**75D**	**50D**	**25D**	**10D**
3-Months										
mean	0.920	0.962	1.001	1.038	1.076	0.083	0.047	0.021	0.008	0.003
median	0.921	0.962	1.001	1.037	1.071	0.082	0.047	0.021	0.008	0.002
std.dev.	0.024	0.011	0.002	0.014	0.029	0.025	0.014	0.006	0.003	0.001
skew	-0.554	-0.528	-1.003	1.178	1.438	0.570	0.602	0.738	0.979	1.200
kurt	0.044	0.201	1.771	1.781	2.669	-0.031	0.176	0.711	1.399	1.963
6-Months										
mean	0.885	0.946	1.003	1.056	1.113	0.119	0.067	0.030	0.011	0.004
median	0.884	0.945	1.003	1.056	1.107	0.121	0.068	0.031	0.011	0.004
std.dev.	0.031	0.014	0.004	0.019	0.041	0.032	0.018	0.008	0.003	0.001
skew	-0.229	-0.136	-1.161	0.869	1.183	0.276	0.280	0.385	0.645	0.893
kurt	-0.675	-0.613	1.473	0.612	1.564	-0.751	-0.634	-0.245	0.380	0.885
9-Months										
mean	0.859	0.934	1.004	1.071	1.144	0.145	0.081	0.037	0.014	0.005
median	0.858	0.932	1.005	1.071	1.137	0.148	0.084	0.038	0.014	0.005
std.dev.	0.034	0.016	0.005	0.023	0.051	0.035	0.020	0.009	0.004	0.001
skew	-0.108	0.031	-1.106	0.749	1.128	0.180	0.168	0.255	0.523	0.794
kurt	-0.882	-0.894	1.183	0.205	1.269	-0.962	-0.898	-0.580	0.042	0.552
1-Year										
mean	0.838	0.925	1.006	1.084	1.171	0.166	0.093	0.042	0.016	0.005
median	0.837	0.923	1.007	1.084	1.165	0.170	0.096	0.044	0.016	0.005
std.dev.	0.037	0.017	0.007	0.026	0.059	0.038	0.021	0.010	0.004	0.002
skew	-0.045	0.121	-1.044	0.660	1.087	0.135	0.107	0.171	0.443	0.726
kurt	-1.005	-1.043	0.930	-0.051	1.051	-1.075	-1.046	-0.794	-0.187	0.321

Hence, this is a recursive equation where the i-th moment is dependent upon all the previous moments. Note that when $i = 0$, the function yields the probability function as follows:

$$\mathbb{E}[X^0] = 1 = \sum\nolimits_{j=0}^{n} a_j \left[K_{j+1} - K_{j+1}\right] \tag{10}$$

which proves that the probability masses sum to 1.

One empirical problem in retrieving the RND is that it is inevitable to have negative probabilities. Malz (2014) has an excellent demonstration as how inevitable this can happen. Cincibuch (2004) reaches a similar conclusion (called ghost-smile). Burgin and Meissner (2012) argue that the risk-neutral probabilities can be negative as they are “pseudo” probabilities which have no meaning except for computations of derivative prices. Haug (2004) also acknowledges this problem in a much broader context.

Theoretically, negative probabilities seem impossible. Yet, in the mathematical literature, negative probabilities are well understood. In a pseudo way, negative probabilities are used for the facilitation of computations. According to Wikipedia, “In 1942, Paul Dirac wrote a paper ‘The Physical Interpretation of Quantum Mechanics’ where he introduced the concept of negative energies and negative probabilities: ‘Negative energies and probabilities should not be considered as nonsense. They are well-defined concepts mathematically, like a negative of money.’”

In a sate price theory, Arrow-Debru prices are similar to discounted risk-neutral probabilities. As a result, risk-neutral probabilities must be non-negative when the discount rate is a constant. If discount rates are stochastic (i.e. state-dependent discount rates), then a negative probability for a particular state must be “balanced” by a negative discount rate of the same state in order to preserve a positive Arrow-Debru price of the state (in order to avoid arbitrage). In other words, negative risk-neutral probabilities can exist as long as discount rates are stochastic.

As a result, there is no theoretical reason that an RND cannot have negative densities (provided that discounts are negative in the corresponding states). Negative discount rates represent negative risk premiums (assuming non-negative risk-free rate). In our empirical results, we discover that in some periods probabilities are so negative that the second and fourth moments are negative. This indicates that most of the states have negative risk premiums in these periods. To be clear later in our empirical results, the second moment is negative during the pre-Lehman crisis (real estate bubble) period and the negative fourth moment occurs in the pre-Lehman crisis period and the pre-european crisis period. These periods enjoy super prosperity (bubble) and then followed by crashes. In summary, the moments generated from RND have implications on risk premiums.[17]

Figure 2 plots various moments of the RND. Panels A, B, and C plot un-centralized variance, skewness and kurtosis of the piece-wise flat RND respectively. We choose to use the un-centralized moments as opposed to the usual centralized moments (which are variance, skewness, and kurtosis) in that these are straightforward power functions of the underlying variable as opposed to a mixture of various powers of the underlying variable. As a result, the information carried in these un-centralized moments is much cleaner than the usual centralized moments.

We see that the term structure the moments (M2 $\sim$ M4) co-vary with the swap curve (Panel D) amazingly well. This is surprising in that these are two distinctly separate markets. Yet option investors form their subjective RNDs in the same manner as the FX swap investors. As the second (M2) and fourth (M4) moments move down (or third moment (M3) moves up), the swap spreads move down.

[Figure 2 and Table 3 Here]

Table 3 summaries Figure 2 quantitatively, as we can see different moments of the RNDs of different maturities are highly correlated. They are also individually highly correlated with FX swap spreads. We note that skewness is the most highly correlated moment with the FX swap spread. Other than 1W and 2W, the skewness dominates other moments in explaining FX swap spreads at all the other tenors. The correlation of skewness and FX swap spread is generally negative (with the only exception of 1W but the magnitude is so small that it is insignificantly different from zero). Also interesting is the kurtosis correlation with the FX swap spread. The correlation between kurtosis and FX swap spread is only slightly less than that between skewness and FX swap spread, and the magnitude is positive.

The second and fourth moments, like variance and kurtosis, are generally regarded as market

[17] As a robust check, we also fit a piece-wise linear RND and a cubic-spline RND. When we replace the piece-wise flat RND with a piece-wise linear RND or cubic-spline RND (both are described in the Appendix), the problem of negative probabilities alleviates (but not totally resolved); and yet two other problems emerge. First, the patterns of the moments we observe in piece-wise RND disappear and second, the shape of RND becomes unstable (change radically), which is the evidence of over-fitting. As a result, it seems to be the tradeoff between potentially under-fitting (polynomial with power of 0 – piece-wise flat) which results in more negative probabilities but stable and over-fitting (polynomial with power of 1 – piece-wise linear and polynomial with power of 3 – cubic spline) which results in fewer negative probabilities but unstable. As noted, empirically, the behavior of piece-wise flat performs best. The results of the robust check are available upon request.

Table 3: Correlation Results

(a) Correlations among Moments

tenor	1W	2W	1M	2M	3M	6M	9M	1Y
varn. vs. skew.	-0.927	-0.92	-0.828	-0.708	-0.649	-0.603	-0.585	-0.567
varn. vs. kurt.	0.865	0.903	0.856	0.791	0.767	0.772	0.795	0.815
skew. vs. kurt.	-0.983	-0.989	-0.989	-0.983	-0.976	-0.96	-0.94	-0.917

(b) Correlations with FX swap spreads

tenor	1W	2W	1M	2M	3M	6M	9M	1Y
variance	-0.168	-0.148	-0.012	0.124	0.215	0.395	0.486	0.536
skewness	0.042	-0.099	-0.447	-0.71	-0.813	-0.912	-0.932	-0.934
kurtosis	-0.097	0.023	0.364	0.626	0.732	0.85	0.882	0.892

and tail risks respectively and the third moment (or skewness) as market sentiment.[18] The negative correlation between the third moment and either the second or fourth moment validates this common wisdom. Given that FX swap spreads usually are also regarded as risk premiums, the high positive correlation between skewness and swap spreads produced by the piece-wise flat RND is stimulating.

3.3 Information Contents of RND

Ever since Chiras and Manaster (1978) and Figlewski and Canina (1993), studies of the forward-looking density parameters have attracted academic and practical attentions. Instead of studying the implied volatility that is based upon the Black-Scholes model, the entire density function has been retrieved to study the risk and behavior of the underlying asset. In other words, RND has replaced the implied volatility to retrieve information from the market. The disadvantage of using the implied volatility is that it absorbs all information about the underlying asset into one single parameter. On the contrary, more detailed information can be learned from RND. In particular, the second moment is risk and the third moment is generally regarded as risk

[18]The third moment (or skewness) under the real measure is regarded as risk premium (in other words, even there is no skewness under the risk-neutral measure, under the real measure there would be skewness due to risk aversion – investors hate losses more than they like gains). Under the risk-neutral measure, it does not represent risk premium but indicates how future movements are likely to be.

premium. The fourth moment is tail risk which can be regarded as a fear index similar to the VIX index. In a way, we can regard RND as decomposition of the implied volatility into finer categories.

3.3.1 Risk Premium

Breuer (2011) argues that the volatility of (implied) volatility connects closely to risk premium. Given that this measure is not contemporaneous (i.e. cannot be measured synchronously with the volatility), we use skewness as an alternative proxy. It has been recognized that the volatility of volatility can drastically skew the distribution of the underlying asset. That is, even if the distribution of the spot is symmetric (i.e. zero skew), stochastic volatility can result in skewness for the underlying asset under the risk neutral measure. It is then robust to use the risk-neutral skewness as an alternative proxy to evaluate risk premiums.

Risk premiums of foreign exchange (FX) rates can be directly observed from the FX swap spreads.[19] The basic intuition is that banks use FX swaps as a tool to borrow either U.S. dollars when facing funding difficulties. Therefore, assuming that the financing liquidity (supply) is stable at the overnight interest rate market, a surge or drop of this spread is a consequence of the change of perceived risk (and hence risk premium). For example, if financial institutions need USD for financing, they sell the euro spot and buy the forward. Through the transaction, the demand of emergent funding leads to a higher EUR/USD forward price than the spot price and leads to a positive spread (forward minus spot). The results are reported in Table 4.[20]

There are 8 panels in Table 4, from 1W to 1Y, for 8 options with different maturities of any given day. Throughout all tenors, M3 is consistently significantly negative on the FX swap spreads. Yet, the significance is more pronounced for shorter tenors (t statistics are -13.99, -17.5,

[19]Literature has documented that FX swap spreads and violation of related parity condition have been used as a measure for the credit risk premium, i.e., Baba and Packer (2009). The swap spread here is calculated or approximated as $\ln F - \ln S$ where F and S represent forward and spot prices respectively. Under no arbitrage, it should equal $r_d - r_f$ where r_d and r_f represent domestic (US) and foreign (euro) interest rates respectively that are used in calculating RNDs. Hence, a violation of the no arbitrage condition is regarded as a credit risk premium. However, when interest rates are stochastic, $\ln F - \ln S$ does not necessarily equal $r_d - r_f$ due to interest rate risk premium. As a result, the two types of risk premiums are hard to distinguish from each other. For this reason, we use the whole swap spreads in our regressions. As a robust check, we also run regressions over spreads that are adjusted for $r_d - r_f$. Our result shows that M3 and M4 continue to be significantly and negatively correlated with the adjusted spread for all tenors. The regression result is available upon request. There are market participants who trade on such risk premiums, known as the carry trade.

[20]The reported t statistics are adjusted by Newey-West variance-covariance matrix estimation.

and -12.37 for 1W, 2W, and 1M respectively) than for longer tenors (t statistics are smaller, between -3.49 and -5.60). Furthermore, Figure 2 confirms that the term structure of M3's moves consistently with the term slope of the spread curve. When the spread curve is upward sloping (e.g. spot > forward, mainly starting 2012 till recent), M3 is negative and when it is downward sloping (e,g, spot < forward at 2008), M3 is positive.

Table 4: FX Swap Spread Regression

SPRD 1W	Coef	t stat
a	-3.655	-9.23
M2	-6526.9	-3.23
M3	-35335	-10.58
M4	-20836	-13.99
IV	-892.41	-0.28
#	1847	
adj. R2	64.38%	

SPRD 2W	Coef	t stat
a	-6.4124	-12.94
M2	2849.23	0.84
M3	-31156	-12.95
M4	-24511	-17.5
IV	-13010	-2.32
#	1847	
adj. R2	72.25%	

SPRD 1M	Coef	t stat
a	-11.007	-16.2
M2	-943.2	-0.61
M3	-24640	-12.37
M4	-14732	-10.06
IV	-4371.1	-1.81
#	1847	
adj. R2	72.26%	

SPRD 2M	Coef	t stat
a	-12.74	-12.62
M2	-4802.1	-4.06
M3	-10308	-5.6
M4	1162.28	0.76
IV	2630.71	1.39
#	1847	
adj. R2	79.25%	

SPRD 3M	Coef	t stat
a	-11.308	-8.63
M2	-5750.7	-5.92
M3	-4771.9	-4.17
M4	6333.36	5.58
IV	4247.24	2.71
#	1847	
adj. R2	83.86%	

SPRD 6M	Coef	t stat
a	-9.5894	-5.18
M2	-3025.9	-3.79
M3	-1818.8	-3.59
M4	6275.98	8.95
IV	364.549	0.29
#	1847	
adj. R2	89.21%	

SPRD 9M	Coef	t stat
a	-5.8833	-2.23
M2	-1739.5	-2.15
M3	-1310.7	-3.49
M4	5424.03	8.24
IV	-1275.4	-1.01
#	1847	
adj. R2	90.43%	

SPRD 1Y	Coef	t stat
a	-0.8901	-0.25
M2	-542.13	-0.68
M3	-1241.9	-4.02
M4	4415.84	7.14
IV	-2833.6	-2.34
#	1847	
adj. R2	90.83%	

The decrease of the significance of M3 (as the tenor increases) indicates that the "skew risk" is more of a short term phenomenon. For long horizons, "tail risk" takes over and becomes more dominant. We note that, for shorter tenors, M4 has roughly the same significance as that of M3 and yet for longer tenors M4 dominates M3. Given that that rare events are priced by M4, our results indicate that they are more priced in longer terms than in shorter terms, as in the short run the anticipation/likelihood of rare events is small. This is consistent with the rare event premium identified by Liu, Pan, and Wang (2005) and the disaster risk premium by Barro (2009).

We notice that for the three shortest tenors, M4s are negatively significant; but for longer tenors M4s are positively significant. We hence hypothesize a "clientile effect" in the FX market. For the short run, financial firms trade for short term profits but in the longer term, industrial firms hedge their foreign currency exposures with deep out-of-the-money (OTM) options. As a result, M4 carries the same sign as M3 for short tenors (1W, 2W, and 1M) to reflect the risk premiums born by financial firms. Yet for long tenors (2M $\sim$ 1Y), as industrial firms dominate in hedging their currency exposures, the sign depends on net imports to exports of the U.S. with respect to the E.U.

To support our hypothesis of the "clientile effect", we present Figure 3 which demonstrates that the U.S. always imports more than exports to the E.U. and the average net importing amount is about \$10 billion every month.[21] This indicates that E.U. exporters are exposed to USD currency risk and shall purchase OTM options to hedge. As the OTM options regard only tail risk, this strong hedge force from the E.U. turns the sign of M4 from negative to positive.[22,23] That is, for long-term tenors, the positive spread (forward minus spot) implies higher future EUR/USD exchange rate, positively correlating to hedge behavior of E.U. exporters.

To measure this "clientile effect", we regress the changes of monthly M4s for each tenor over the net changes of the U.S. imports from the E.U. (controlling for lags (lag1 $\sim$ lag4)) and over

[21] Trade in Goods with European Union: https://www.census.gov/foreign-trade/balance.

[22] As the LC (letter of credit) will be cashed into the account in 30, 60 and 90 days, those companies tend to hedge with corresponding tenors, i.e, 1M, 2M, 3M and 6M. Galat and Schreiber (2013) uses a unique database to show that the corporate tends to take long position of deep OTM options with the average of 97 days to expiration, while finance firms tend to trade short length contracts, i.e, the average is 40 days, without strong preferences over long/short positions. All of these indicate, in contrast to 1W or 2W tenor, longer tenors are traded more by industrial companies.

[23] To our knowledge, European exporters either buy 1M-6M EUR/USD forward or buy OTM call options to hedge their USD exposure, and a symmetric (opposite) hedge is performed by their counterparties (U.S. importers), causing M4 to change signs.

the changes of spreads of corresponding tenors.[24] We show that the demand for future exports has a strong impact on options but less so on FX swap spreads. The results are listed in Table 5. As we can see, the future net exports to the U.S. affects M4 much more than past net exports, and 1 to 3 month ahead exports most significantly influence today's M4. However, in panel (b) of Table 5, we do not find net exports from the E.U. to significantly affect the spreads. These results indicate that, for longer tenors and positive FX swap spread, the demand of hedging from industrial companies can also drive M4s higher.

Our hypothesis of the "clientile effect" is similar to Han (2008) who uses sentiment to explain the volatility smile. Our RND results confirm that different risk attitudes between financial firms and industrial firms would result in different behaviors of RND moments at different tenors. In the case of EUR/USD exchange rates, longer tenor moments are informative in explaining currency exposures of exports and imports while shorter tenor moments help explain risk premiums better.

[Figure 3 Here]

[Table 5 Here]

Our results also shed light on volatility/variance risk premiums. Carr and Wu (2009) demonstrate that riskier stocks (i.e., higher beta and smaller size stocks) commonly have higher volatility risk premiums. They argue that investors of those stocks want to pay a higher premium to hedge an adverse stock movement, implying that the volatility risk tends to positively correlate with stock returns. Bollerslev, Tauchen and Zhou (2009) empirically show a positive relation between stock returns and variance risk premiums.[25] We argue that M4 (kurtosis) and M2 (volatility) increase the risk premium for people who consider holding euros as an investment asset, and higher M2 and/or M4 lead to a negative swap spread. Symmetrically, people who consider holding U.S. dollars as their investment also require risk premiums for holding U.S. dollars, resulting in positive swap spread.

It is interesting to note that the Black-Scholes implied variance (IV) performs poorly in explaining swap spreads. The coefficients are only marginally significant. This indicates that RND moments can explain risk premiums better than the implied variance. Although some

[24] Given that net E.U. export data is monthly, we average daily M4s and spreads to calculate monthly.

[25] Also the implied volatility.

regression coefficients for the IV are significant, the significance is marginal and only a fraction of those of M3 and M4. For example, the most dominant case is 1W spread where the t statistic for the IV is -0.28 and yet the t statistics for M2 $\sim$ M4 are -3.23, -10.58, -13.99 respectively. Similar dominance can be found in the case of 6M spread. It is clear that RND moments carry much more valuable information than the IV.

Finally, we note that R-squares are increasing with tenor. This is perhaps the most interesting result (somewhat surprising) which could be due to the fact that trading behavior of exporters who are not sensitive to short term market changes (hence less volatile). The R-squares are strictly monotonic increasing from 64.38% for the 1W regression to as high as 90.83% for the 1Y regression. In the case of 1Y regression, M4 is the most powerful explanatory regressor (t statistic is 7.14), then M3 (t statistic is -4.02), and finally IV (t statistic is -5.80).[26]

In conclusion, by using FX RND higher order moments, we show FX options market contains richer economic information. Unlike Carr and Wu (2009), Bollerslev, Tauchen and Zhou (2009), and Kozhan, Neuberger, and Schneider (2013) who clearly define risk premiums under a specific moment, we study the intricate relationship between M3 and M4 and their impacts on risk premiums (proxied by FX swap spreads). Our results are astounding in that both M3 and M4 explain FX swap spreads, but at different magnitudes at different tenors. We also find M4 to switch signs which is consistent with the hypothesis of the export/important relationship between the U.S. and the E.U.

3.3.2 Prediction of Future Variance, Future Exchange Rate, Economic Policy Uncertainty, and Currency Influence index

As mentioned in the Introduction, RND has been proven effective in predicting (1) future underlying price movements; (2) future option prices; (3) future realized volatility/variance; and (4) future distributions of the underlying assets. Yet, this evidence has been predominantly on stock indexes (e.g. Gemmill and Saflekos (2000), Khrapov (2014), Jiang and Tian (2005), and numerous others.

Some limited evidence has been provided over currencies. Using earlier data, Christensen and Prabhala (1998), Jorion (1995), and and Xu and Taylor (1994) study volatility forecasts. In addition to forecasting volatility, Christoffersen and Mazzotta (2005) also forecast density for

[26] M2 is not significant (t statistic is -0.68).

the period from March 31, 1992 to February 19, 2003. More recently, Chen and Gwati (2012) who predict underlying FX rates from January 1, 2007 to April 19 2011.

Our study adds to the literature in the following obvious ways. We discover that RND contains a lot more prediction power than future volatility and underlying asset price movements. We confirm the prediction power of the literature in forecasting volatility and underlying price movements. In addition, we find that RND can be used to predict macro environment that implied volatility cannot. For example, RND leads economic policy uncertainty (Baker, Bloom, and Davisc (2016)) and currency influence index (Alter and Beyer (2014)) which in turn lead IV (implied variance).

Prediction of Underlying Price Movements

We first examine if RND can predict future movements of the underlying asset. To test this hypothesis, we run regressions of the changes of exchange rates on the changes of explanatory moments (M2 $\sim$ M4) and implied variance (IV). For example, we let the changes of 1W moments and IVs at time t predict the exchange rate changes at $t+1$. The results are reported in Table 6.

[Table 6 Here]

First of all, we report that RND moments always provide supplemental predicting powers to a time-series forecasting model. As we find out that the EUR/USD exchange rate demonstrates strong return auto-correlation (i.e. highly significant Lag1_delta_τ_spot coefficients for various tenors where τ represents a tenor), we show, after controlling the lagged 1 $\sim$ 4 spot returns, RND moments are always significant in predicting future FX returns.

Horse-racing with IV, RND moments dominate IV for short tenors (1W, 2W, 1M and 2M). Take 1W and 2W as examples, the t statistics for IV are 6.13 (1W) and 1.40 (2W) and those for M2 $\sim$ M4 are -6.38, -6.82, and -6.86 respectively for 1W and -0.56, -11.69, and -11.55 respectively for 2W. As pointed out just now, M3 (risk premium) and M4 (tail risk) all have negative coefficients which are consistent with the hypothesis that . (Negative skew lead to higher risk and positive return.) Yet IV has a positive coefficient and significant.

We focus our analysis particularly on M3. We note that M3 is consistently significantly negative throughout all tenors whereas coefficients for M2 and M4 are not so (M2 coefficients are generally significantly negative but not for 1M which is significantly positive and 2M which is positive but insignificant. M4 coefficients are significantly negative for short tenors 1W $\sim$ 3M but turn significantly positive for long tenors 6M $\sim$ 1Y.[27][28]) On the contrary, we find that the coefficients for IV are significantly positive in nearly all cases (except for 2W). We can see that M3 continues to outperform other explanatory factors (M2 and IV in particular), which is same as the previous subsection.

Further examination of the results reveals that while M3 is statistically significant for all tenors, it dominates IV and other moments for short tenors (1W $\sim$ 2M) but is dominated by IV and M2 for longer tenors (3M $\sim$ 1Y). We also discover that M4 behaves closely to M3 and is more important to explain short-term exchange rate changes. The t statistics for M3 and M4 (compared to those of M2 and IV) are -6.82 and -6.86 (compared to -6.38 and 6.13) for 1W; -11.69 and -11.55 (compared to -0.56 and 1.4) for 2W; -9.66 and -11.05 (compared to 3.15 and 2.0) for 1M; -9.93 and -9.05 (compared to 0.71 and 4.78) for 2M. On the contrary, for long tenors, 2M and IV are more significant in predicting future exchange rate changes (-2.56 and 6.65 compared to -3.07 and -2.24 for 3M; -11.81 and 12.46 compared to -4.85 and 2.68 for 6M; -15.02 and 17.45 compared to -5.11 and 3.09 for 9M; -16.61 and 20.35 compared to -7.46 and 5.46 for 1Y).

It is understandable that the significance of M3 is decreasing with tenors (that is prediction power is weaker as prediction horizon becomes longer.) This is because those sudden (symmetric or asymmetric) discontinuities of trades (jump) that cause skewness become a relatively minor issue in the long run.

Prediction of Macro Environment

Different from other assets, exchange rates reflect relative economic strengths of two nations. As a result, RND moments implied by FX options should carry information about the macroeconomic conditions of the two nations. In this sub-section, we investigate if RND carries information of two important macro indexes: economic policy uncertainty (EPU) index and exchange rate spillover effect (USD influence index). The EPU index is proposed by Baker, Bloom, and Davisc (2016)[29] who use newspaper coverage frequency to measure the policy-related econo-

[27]This could be resulted from exporters behavior. As ordering increases, part of exporters buy OTM call EUR/USD options to hedge which leads to a increases of M4, while other exporters who do not hedge just sell dollar and buy euro once the LC is eligible to cash in. The increased demand for euros drives up EURO/Dollar exchange rate.

[28]This is consistent with the clientile effect examined earlier.

[29]They have a web site to share the EPU index. (http://www.policyuncertainty.com/index.html)

mic uncertainty. If the FX options market is efficient, then those implied moments should reflect macroeconomic issues. We are also interested in knowing if RND can predict the spillover effect of USD to other countries' exchange rates. Alter and Beyer (2014) use the VAR (Vector Auto Regression) impulse response function to quantify spillovers between sovereign credit markets and banks in the E.U. Given option prices contain forward looking information, we expect the implied moments to be highly correlated with those two indexes and can even forecast both indexes.

Figure 4 presents the EPU index, IV and different implied moments in our sample period. As observed, the index is positively correlated with IV, M2, M4 and negatively correlated with M3.

[Figure 4 Here]

Using the Granger causality test, we examine the prediction ability between RND moments and the EUP index. We test if the EPU index can be forecasted by following groups: (1) term structure of IVs (2) term structure of M2s (3) term structure of M3s (4) term structure of M4s (5) 1-week moments (6) 2-week moments (7) 1-month moments (8) 2-month moments (9) 3-month moments, respectively. And, we also test reversely if those groups can be forecasted by EPU index.[30] The results are summarized in Table 7.

[Table 7 Here]

We discover that the term structures of M2s, M3s, and M4s can predict (Granger-cause) the EPU index, but not vice versa. On the contrary, the term structures of IVs is Ganger-caused by the EPU index. In other words, IV is not a good predictor (worse than RND moments) of the EPU index.

Within the RND moments, we find that M3s and M4s have larger powers than M2s do in rejecting the hypothesis that the EPU index doesn't have an influence. Furthermore, RND moments of shorter tenors (e.g., 1-week, 2-week and 3-week) predict the EPU index better than moments of longer term tenors. This indicates that the EPU index is more related to short-term

[30]We run Granger test from lag 1 to lag 10. The results are similar in the sense that the RND tends to have prediction power over EPU and influence indexes, but the reverse is generally not true. Here we only show the results of lag 6 for the EPU index and lag 1 for the Influence Index.

economy turbulence. As mentioned earlier, the reverse does not hold – no evidence supports that the EPU index can predict RND moments of any particular tenor.

For the spillover effect, we use the USD influence index constructed by Zhou, Wang and Cheng (2016) who adopt the methodology by Alter and Beyer (2014). The following currencies from the developed countries are chosen in our sample: US dollar, European euro, Japanese yen, British pound, Swiss francs, Canadian dollar, Australian dollar, Chinese yuan, and Hongkong dollar. We run VAR for the exchange rates of U.S. dollar against these currencies.[31] The USD influence index is created by adding up all the response functions in the VAR.

A higher spillover effect means that the changes of the USD exchange rates with the chosen countries influence more strongly the changes of the exchange rates among these countries. The higher influence of one country over other countries implies lower effectiveness of diversification. As a result, the hedge demand and hedge cost increase.[32]

In Figure 5, we show the USD influence index and 1-month M4. We find a one-time increases at 4Q 2008 for U.S. exchange rate influence index, and their co-movements still can be observed after 2008. We take the first order difference of the spillover index to remove the one time fixed effect and show the change of the index in the bottom chart.

[Figure 5 Here]

As reported in Table 8, the Granger causality test results reveal that, similar to the EPU index, the term structures of M2s, M3s, and M4s can predict the USD influence index; but not reversely. Also similar to the results of the EPU index, the term structure of IVs is affected by the USD influence index. This indicates that RND moments, but not IV, are capable of forecasting the USD influence Index. RND moments can predict the USD influence index; yet interestingly, for longer tenors (e.g., 3-month, 6-month, 9-month and 1-year), supported by the rejection of the null hypothesis that the USD influence index cannot predict RND moments (at increasing probabilities as tenors get longer). This implies that the change of the USD influence index is more related to long-term fundamental issues.

[Table 8 Here]

[31]The details of the VAR are available from the authors upon request.

[32]The diversification issues on options pricing are discussed extensively by Jarrow, Lando and Yu (2005) and Amin (1993).

In summary, using Granger causality test, we discover that RND moments have prediction powers of both EPU and USD influence indexes; yet the reverse is not true. On the contrary, both indexes Granger-cause IV but the reverse is not true. Furthermore, short term RND moments have higher a power to predict the EPU index and long term moments have a higher power to predict the USD influence index.

Prediction of Future Volatility

It is a popular exercise to see if the implied volatility carries any prediction power of future realized volatility. The literature on information content in implied volatility postulates that implied volatility, since it is computed off option prices, contains forward-looking information and hence should be a good predictor of future realized volatility. Also in the literature is that under stochastic volatility, implied volatility also carries volatility risk premium and hence should be higher than realized volatility.

Prediction of realized volatility is analyzed via a set of regressions of various predictors on the realized volatility. In each regression, the realized volatility is the dependent variable and the independent variables are: Black-implied volatility and the second through fourth moments of RND using equation (9). The results are presented in Table 9.

[Table 9 Here]

In order to match the tenor of the implied volatility or an RND moment, each realized volatility is computed using the hold period equal to the corresponding tenor. For example, in the 1W regression, since the volatility is forward-looking for the next 5 business days, the realized volatility is calculated 5 business days after the date of the observed implied volatility. Same applies to other tenors. Also, the realized variances (RZ_VAR) are computed using exchange rates (level) in order to be comparable to the RND moments that are also computed off exchange rate levels. For the Black's implied variance (IV) to also be comparable, we must do the following adjustment. Note that IV is computed under the assumption of log-normally distributed underlying exchange rates, and hence the adjustment is computed as follows:

$$\zeta_\tau^2 = \sigma_\tau^2 \bar{S}_\tau^2$$

where σ_τ is the Black's volatility over the same horizon τ as that of the RND and $\bar{S}_\tau$ represents the average of the exchange rate levels over the horizon τ, and τ is from 1W to 1Y (8 maturities) as indicated in Table 9.

Once again, Black's implied variance (after adjustment by the level) is dominated by the moments. However, interestingly it is not dominated by M2 but rather M4. Out of 8 cases, IV is dominated by M4 in 5 cases (1W, 2W, 3M, 6M, and 9M); by M3 in 4 cases (1W, 2W, 6M, and 1Y); and by M2 in 3 cases (1W, 6M, and 9M).

In general, M4 is the best predictor (significance in 7 out of the 8 cases), followed by M3 (6 cases) and IV (6 cases). Interestingly M2 is the worst performer (significance in 4 out of 8 cases). We note that the moments are non-centralized and hence are not impacted by the expected value of the underlying exchange rate (while IV and the realized variance are).

The power of prediction (measured by adjusted R-squares) increases as the time horizon increases (except for 1Y which is less than 9M, but still greater than 6M). This indicates that short term noises reduce the prediction power. As longer horizons are considered, noises are averaged out, prediction powers improve.

Finally, a minor but worthy point is that M4 signs are positive for short horizons but negative for long horizons. Again, this is consistent with the "clientile effect" discussed in a previous subsection (also footnotes 27 and 28). Other predictors have changed signs quite randomly.

3.3.3 Explanatory Power of Major Events

In this section, we follow the literature and examine how our results could provide insights on major events in the Euro zone. Gabaix et. al. (2016) uncover that since the Fall of 2008, "crash risk" has increased dramatically, implied by the FX options data.

We compare the moments of the RND with the Black-implied variance (IV) on major economic events. As a benchmark, we also present the VIX series as the VIX index is an indicator of "fear" and reacts sensitively to major economic events. VIX and IV are plotted in Figure 6. In each panel, there are 8 time series corresponding to 8 maturities.

[Figure 6 Here]

Similar to Figure 2, in each panel, we use shaded areas to denote the time windows from pre-crisis to middle-crisis. The first shaded area refers to the 2008 Subprime Crisis starting from the Bear Stearns event[33] to the Citi rescue plan.[34] The second shaded area is the European Sovereign Crisis, as we use the date of April 6, 2011, when Portugal asked for financing help to denote the beginning of the crisis. We use the October 21 announcement date for Greek second bailout plan from the E.U. as the last day of second shaded area. Finally, the Flash Crash happened in May 6, 2010, which is marked by a dark vertical line between two crises.

We first compare IVs from Figure 6 with M2s from Figure 2, we see close co-movements between the two sets of series. Both sets of series are high at the Subprime Crisis (2008-9), Flash Crash (2010), and the European Sovereign Crisis (2011-2). Finally, they both start to increase at the beginning of 2015. However, there are notable differences between the two. M2s of various tenors reach their peaks at the European Sovereign Crisis (2011) but are very closely followed by the Flash Crash (2010) and the Subprime Crisis (2008). On the contrary, IVs reach their peaks at the Subprime Crisis and are mild during the Flash Crash and the European Sovereign Crisis (this is because the levels remain high throughout the entire period). Furthermore, M2s during the pre-Subprime-Crisis period and the pre-European-Sovereign-Crisis period are either close to 0 or barely positive but the IVs are very high. This result indicates that the second moment of the RND is more sensitive to drastic events than Black-implied variances.

While disastrous events are easy to spot visually graphically (as in Figures 2 and 6), we need to further quantify them so that we can examine if the moments from the RND have any explanatory powers of rare events. To do that, we use the VIX index as a proxy for large events. We provide two sets of regression results. The first one is $\mathrm{VIX}(t)$ on $\mathrm{M}(i,t)$ where $i = 2, 3, 4$ representing the i-th moment; and the second one $\mathrm{VIX}(t)$ on $\mathrm{M}(i, t-22)$ which the VIX index is approximately lagged by one month (we use 22 observations to approximate a month). In both sets of regressions, the Black-Scholes implied variance (IV) is used as a control variable.

The results are summarized in Table 10. We find that lagged regressions have more significant moments than concurrent regressions, indicating that moments lead the VIX index and hence have predicted powers of large events (proxied by the VIX index). This is true for all tenors, even after controlling for the implied variance (IV). For example, for the 1-week tenor, the t statistics of M2 $\sim$ M4 are 3.66, 3.83, and 4.07 respectively in the concurrent regression versus 9.45, 9.92, and 9.81 respectively in the lagged regression. Similar observations are made for

[33]On March 14, 2008, Bear Stearns' shares plummeted, and it was quickly acquired in two days by JPMorgan Chase for 2 dollars per share.

[34]At October 3, 2008, the Fed initiated a \$700 billion TARP, Troubled Assets Relief Program to purchase failing bank assets, plan, and gave out 33.6 billion to 21 banks in the second round of disbursements. Later on November 24, the U.S. government agreed to inject another 20 billion of capital into Citigroup.

other tenors. We also find that the significance of the implied variance is weaker in the lagged regressions than in the concurrent regressions, which suggests that moments are less influenced by IV in the lagged regressions.

Furthermore, we note that in general the significance levels are higher for longer tenors than in shorter tenors. This could be due to the effect that the moments of shorter tenors tend to be more noisy and hence carry less information (larger standard errors).

[Table 10 Here]

3.3.4 Conclusion

Our main focus of this section is the explanatory powers of the M3s and M4s. In contrast to the M2s, the M3s and M4s do not have any change for the Flash Crash crisis. This is interesting in that Flash Crash is not considered as a tail risk event (M4), nor does it add any risk premium (M3), by investors. As a consequence, it is not priced. In hindsight, this result is amazingly accurate as Flash Crash has hardly any impact on the economy.

One particularly interesting and insightful result is that the M3s present enormous negative risk premium during the pre-crisis periods – reflecting excess risk taking during the bubble periods (Subprime and European crises). It is more so for the real estate bubble before the Subprime crisis than the period before the European Sovereign crisis, as the real estate bubble is more severe. Similarly, M3 reacts more severely to the European Sovereign crisis than to the Subprime crisis as our underlying asset is the EUR/USD exchange rate. Also note that this behavior of M3 is quite different from M2 (or IV or VIX), as it should.

Another interesting observation of M3 is that it does not become negative (i.e. positive risk premium) right at the crisis but lag by a few months. And also note that, during the Subprime Crisis, long and short tenors are drastically different – long M3s are positive but short dated M3s are negative – meaning that for the short term investors charge high risk premium but long term investors are still optimistic.

M4 tells a similar story (but simpler) to M3 that there are two high tail risk events – Subprime Crisis and European Sovereign Crises. Also before each crisis there is a negative risk build-up.

Our results on M3 and M4 shed some light on the rare event premium by Liu, Pan, and Wang (2005).

Lastly, we examine the term structures of the RND moments. The term structures of M2s $\sim$ M4s are either negatively sloped or flat prior to the Subprime Crisis and European Sovereign Crisis and not Flash Crash. This indicates that the moments of the RND provide a warning signal for a major economic event, and is an ex-ante measure for the expected market turmoil.

In summary, we find that the moments from the RND carry much more, better, and subtle information than IV and VIX. Especially we find that Flash Crash is not an event by the moments of the RND but both IV and VIX spiked. Also the shape of the term structure of moments seems to provide a warning signal of a major economic event.

3.4 Comparison to Parametric Models

RND is generally regarded as a non-parametric method (if ignoring the choice of a polynomial function) and hence is model-free. In this section, we compare some implications from RND with those of parametric models, mainly the classical Black-Scholes model and the Heston model (stochastic volatility). We also estimate Cox's CEV (constant elasticity of variance) model using RND (as opposed to historical series of the underlying asset). Through these comparisons, we hope to shed light on what is the information gain using RND as opposed to those parametric models.[35]

3.4.1 Term Structure of Volatility

Derivatives pricing professionals have been trying to develop proper pricing models for more than 40 years now. The crucially necessary condition is for the model to simultaneously price all options (cross maturities and strikes) correctly. In other words, the ultimate task for an ideal model is to be able to explain the Black-implied volatility surface.[36] Given that RND automatically price options with different strikes perfectly, we now need to examine how differently RNDs with different maturities imply the volatility term structure.

[35] We note that the Heston model, combined with the smile capability, is proposed by a number of financial companies, such as Bloomberg and Fenics, as the standard model to evaluate currency options.

[36] This is because many options are quoted in Black volatility.

We calculate the expected total variance $\mathbb{E}\left[\int_t^T V(u)du\right]$ for each given maturity. In the Black-Scholes case, it equals $\sigma^2(T-t)$ which is proportional to time to maturity. In addition to the Black-Scholes case, we also look into another popular model – the Heston model with the square root stochastic volatility. The Heston model is described in details in the next section where we estimate the parameters of the Heston model (by minimizing the squared errors of model moments and M2 $\sim$ M4). In the Heston model, the variance follows a square-root, mean-reverting process which has three main parameters reversion speed κ, reversion level θ, and volatility γ. $\mathbb{E}\left[\int_t^T V(u)du\right]$ has a closed-form solution under the Heston model, presented in equation (11). Lastly is the RND-implied term structure. We take the average of each time series (for each maturity) in Figure 2. Given 8 RNDs from different maturities, we have volatility term structure of 8 observation points.

[Figure 7 Here]

We present the results in Figure 7. There are three sets of bars in Figure 7: Black-Scholes (tallest, blue-shaded), Heston (shortest, red-shaded), and RND (red-solid). Each bar is a ratio of the total variance under a chosen maturity over that under the 1-week maturity. Hence the heights of the first bars are identically 1. In the case of the Black-Scholes model, the heights of the second through the eighth bars are proportional to time to maturity, i.e. 2, 4, 8, 12, 26, 39, and 52 for 2W, 1M, 2M, 3M , 6M, and 1Y respectively. However, in the Heston case, the ratios are 1.05 to 1.61 from 2W to 1Y, and the ratios are between 1.45 to 9.31 in the RND case. In other words, the Black-Scholes over-estimates the term structure of variances while the Heston model under-estimates it. In reality, the term structure of variances is somewhere in between.

We note that the Heston model is fitted very poorly by the data. Hence it is not surprising that it also generates very poor result in terms of total variance. It is interesting to see that RND generates very different result than the Gaussian model. This result is robust in the flat and linear cases.

3.4.2 Estimating Variance Elasticity

Britten-Jones and Neuberger (2000) argue that option-implied RNDs embed a rich class of stochastic processes for the volatility. In their work, however, they only explore a simple regime-switching model for the volatility. More accepted by the industry is the Heston model where the variance follows a mean-reverting square-root process. In this section, we estimate a CEV (constant elasticity of variance) model of which the Heston model is a special case.

The empirical work on testing a parametric volatility model with option data is voluminous and not the main focus of our study. Interested readers can see Bates (2003) for an excellent review. In this paper, we are only interested in the CEV parameters, and in particular the mean-reversion parameter implied by the RND.

In the CEV model, when the elasticity parameter β equals 1, it should degenerate to the Heston model. The null hypothesis in our test is hence $\beta = 1$ and the alternative hypothesis is $\beta \neq 1$.

As mentioned, we assume a CEV-volatility model (which is the most flexible diffusion model) as follows:

$$\begin{aligned} dS &= rSdt + \sqrt{V}SdW_S \\ dV &= \kappa(\theta - V)dt + \gamma V^{1/2\beta} dW_V \end{aligned} \tag{11}$$

where $dW_S dW_V = \rho dt$. The density function for the variance is:[37]

$$f(V(s)|V(t)) = yk^{\frac{1}{2-\beta}} \left(\frac{x}{z^{2\beta-1}}\right)^{\frac{1}{2(2-\beta)}} e^{-x-z} I_d[2\sqrt{xz}] \tag{12}$$

where

$$\begin{aligned} k &= \frac{-2\kappa}{\gamma^2(2-\beta)\left(e^{(2-\beta)r(s-t)} - 1\right)} \\ x &= kV(t)^{2-\beta} e^{-(2-\beta)\kappa(s-t)} \\ y &= \mathrm{sgn}[2-\beta](2-\beta) \\ z &= kV(s)^{2-\beta} \\ d &= 2\kappa\theta/\gamma^2 - 1/y \end{aligned}$$

By making the change of variable $w = 2kV(s)^{2-\beta}$, we obtain a non-central chi-square variable with $v = 2d + 2$ degrees of freedom:

[37] See Cox (1975).

$$v = 2 + \frac{4\kappa\theta}{\gamma^2} - \frac{2}{2-\beta} \tag{13}$$

and $\Lambda = x$ degrees of non-centrality:

$$x = V(t)^{2-\beta} e^{-(2-\beta)\kappa(s-t)} \frac{-2\kappa}{\gamma^2(2-\beta)\left(e^{-(2-\beta)\kappa(s-t)} - 1\right)} \tag{14}$$

Hence, the mean and variance (which are $v + \Lambda$ and $2(v + 2\Lambda)$ respectively) are:

$$\begin{aligned}
\mathbb{E}_t[w(s)] &= v + \Lambda \\
&= 2 + \frac{4\kappa\theta}{\gamma^2} - \frac{2}{2-\beta} + V(t)^{2-\beta} e^{-(2-\beta)\kappa(s-t)} \frac{-2\kappa}{\gamma^2(2-\beta)\left(e^{-(2-\beta)\kappa(s-t)} - 1\right)}
\end{aligned} \tag{15}$$

Hence,

$$\begin{aligned}
\mathbb{E}_t[V(s)^{2-\beta}] &= \frac{1}{2k}\mathbb{E}_t[w(s)] \\
&= \frac{1}{2k}\left\{2 + \frac{4\kappa\theta}{\gamma^2} - \frac{2}{2-\beta} + V(t)^{2-\beta} e^{-(2-\beta)\kappa(s-t)} \frac{-2\kappa}{\gamma^2(2-\beta)\left(e^{-(2-\beta)\kappa(s-t)} - 1\right)}\right\}
\end{aligned} \tag{16}$$

The second moment of the volatility can be identified similarly. Under the non-central chi-square distribution, the variance is equal to:

$$\begin{aligned}
\mathbb{V}_t[w(s)] &= 2(v + 2\Lambda) \\
&= 4 + \frac{8\kappa\theta}{\gamma^2} - \frac{4}{2-\beta} + V(t)^{2-\beta} e^{-(2-\beta)\kappa(s-t)} \frac{-8\kappa}{\gamma^2(2-\beta)\left(e^{-(2-\beta)\kappa(s-t)} - 1\right)} \\
&= 4k^2 \mathbb{V}_t[V(s)^{2-\beta}]
\end{aligned} \tag{17}$$

Consequently,

$$\mathbb{V}_t[V(s)^{2-\beta}] = \frac{1}{k^2}\left\{1 + \frac{2\kappa\theta}{\gamma^2} - \frac{1}{2-\beta} + V(t)^{2-\beta}e^{-(2-\beta)\kappa(s-t)}\frac{-2\kappa}{\gamma^2(2-\beta)\left(e^{-(2-\beta)\kappa(s-t)}-1\right)}\right\} \tag{18}$$

Equations (16) and (18) are used in the empirical work for estimating parameters. By setting $\beta = 1$, from (16), we arrive at the mean of the volatility as follows:

$$\begin{aligned}\mathbb{E}_t[V(s)] &= \frac{\gamma^2\left(e^{-\kappa(s-t)}-1\right)}{-2\kappa}\left\{\frac{4\kappa\theta}{\gamma^2} + V(t)e^{-\kappa(s-t)}\frac{-2\kappa}{\gamma^2\left(e^{-\kappa(s-t)}-1\right)}\right\} \\ &= \theta\left(1 - e^{-\kappa(s-t)}\right) + V(t)e^{-\kappa(s-t)}\end{aligned} \tag{19}$$

The estimates under Heston are reported in Table 11. In general, short maturities present stronger speed of reversion κ than longer maturities. This is consistent with the findings in the interest rate literature where short term interest rates present strong mean reversion but not long term interest rates. This could be a result of higher fluctuations in short term variables. Reversion levels θ are similar to the observation in ξ (expected total variance) as expected, as they are mechanically connected in the estimation process. Finally the results of the volatility of variance γ are also expected. Since this parameter is not observable, it is hard to gauge the reasonableness of the magnitudes. Comparing the mean levels of X with the standard deviations of ξ, it would suggest that γ should not be small. In this regard, the estimates seem reasonable.

[Table 11 Here]

From Figure 5, it is suggested that both the Heston model and the Black-Scholes model are likely to be rejected. Similarly, Table 11 also implies that the Heston model cannot explain options cross maturities (i.e. parameters of different maturities are different). As a result, in this section, we make an attempt to estimate the full CEV model.

There exists no easy econometric methodology to estimate the CEV model. As a result, we estimate the parameters by calibrating the total expected variance: $\xi = \mathbb{E}_t\left[\int_t^T V(u)du\right]$. which is computed via Monte Carlo simulations, given that there exists no closed-form solution under the CEV model. There are 8 daily values for ξ (8 maturities) and we simulate corresponding 8 Monte Carlo values. The parameters are solved by minimizing the sum of squared errors.

The number of Monte Carlo paths is 1000. The simulations are based upon the Euler equation: $V_t - V_{t-1} = \kappa(\theta - V_{t-1})\Delta t + \gamma V_{t-1}^{\beta/2}\sqrt{\Delta t}\varepsilon_t$ where $\varepsilon_t \sim N(0,1)$ and Δt is set as 1/252 (daily). For the 1W maturity, there are 5 time steps. For the 2W maturity, there are 10 time steps with the same first five random numbers as used in 1W. The same procedure applies for 1M (21 time steps), 2M (42 time steps), 3M (63 time steps), 6M (126 time steps), 9M (189 time steps), and 1Y (252 time steps). Throughout the entire sample (1847 days), the same seed is used in the simulations.

[Table 12 Here]

The results are reported in Table 12. The β parameter is estimated to be 1.88 which is the average of all daily estimates (with the median to be 1.99). The standard error is 0.38 and hence the average is highly significant. We also obtain results of κ, θ, and γ, which are 1.00, 0.20, and 0.26 respectively with corresponding standard errors of 0.35, 0.13, and 0.14 respectively. With such large standard errors, none of these parameters are significant. The κ estimate is found to be substantially smaller than the Heston result but the γ estimate is larger. As these two parameters balance each other out in explaining the volatility, the results in Table 8 could be unreliable.

4 Conclusion

In this paper, we study the information contents of the risk-neutral density (RND) of EUR/USD foreign exchange (FX) options for the period from January 2, 2008 till March 18, 2015. In particular, we study the four uncentralized moments (M1 $\sim$ M4) of the RND. In conclusion, we discover that higher moments (M3 and M4) have much more explanatory powers over M2 and Black-Scholes implied volatility (or implied variance).

Our empirical findings are in four categories. First, we discover that RND has superior prediction powers of future levels of the underlying asset (which is the EUR/USD exchange rates in our study), confirming the literature on equities. Moreover, the prediction powers of M3 and M4 are higher than Black-Scholes implied volatility.

Second, we discover that RND moments can also predict macro economic variables such as the EPU (economic policy uncertainty) index by Baker, Bloom, and Davisc (2016) and the spillover (influence) index by Alter and Beyer (2014). Interestingly, on the contrary, Black-Scholes implied volatility lags the two indexes.

Third, we discover that RND moments have high explanatory powers of swap spreads which represent risk premiums in the marketplace. We also identify a "clientile effect" in the moments. We discover that M4 has a significantly negative impact on short term spreads but significantly positive impact on long term spreads; whereas M2 and M3 remain the same signs (negative) but the significance deteriorates as tenor lengthens.

Last, in our sample period, we experience three major market events: Lehman crisis in 2008, flash crash in 2010, and European crisis in 2011. We observe that RNDs (via moments) behave drastically differently. In many cases, they behave substantially differently than the implied volatility of the Black-Scholes model. This sheds light on volatility risk and risk premium embedded in FX options.

In addition to RND moments, we also compare RND implied stochastic volatility and parametric stochastic volatility model such as the CEV (constant elasticity of variance) model and the Heston model. We find that the Heston model is rejected. The term structure of variances under the Heston model is too flat compared to the result from the RND. We estimate the elasticity parameter (β) of the CEV model to be quite high (1.88).

References

[1] Alter, Adrian, and Andreas Beyer, 2014, "The dynamics of spillover effects during the European sovereign debt turmoil," Journal of Banking and Finance, 42, 134-153.

[2] Amin, KAUSHIK I., 1993, "Jump Diffusion Option Valuation in Discrete Time," Journal of Finance, 48, 1833-1863.

[3] Baba, Naohiko, and Frank Packer, 2009, "Interpreting Derivations from Covered Interest Parity during the Financial Market Turmoil of 2007-08," Journal of Banking and Finance, 33(11), 1953-62.

[4] Baker, Scott, Nicholas Bloom, and Steven Davisc, 2016, "Measuring Economic Policy Uncertainty," Quarterly Journal of Economics, Forthcoming.

[5] Bakshi, Gurdip, and Dilip Madan, 2000, "Spanning and derivative-security valuation," Journal of Financial Economics, 55(2), 205-238.

[6] Bakshi, Gurdip, and Nikunj Kapadia, 2003, "Delta-hedged Gains and the Negative Volatility Market Risk Premium", Review of Financial Studies, 16, 527-566.

[7] Bakshi, Gurdip, Peter Carr, and Liuren Wu, 2008, "Stochastic risk premiums, stochastic skewness in currency options, and stochastic discount factors in international economies," Journal of Financial Economics, 87, 132-156.

[8] Barro, Robert J., 2009, "Rare Disasters, Asset Prices, and Welfare Costs," American Economic Review, 99(1), 243-264.

[9] Bates, David S., 2003, "Empirical option pricing: a retrospection," Journal of Econometrics, 116, 387-404.

[10] Bliss, R. Robert, and Nikolaos Panigirtzoglou, 2004, "Option-implied risk aversion estimates," Journal of finance, 59(1), 407-446.

[11] Bollerslev, Tim, George Tauchen, and Hao Zhou, 2009, "Expected Stock Returns and Variance Risk Premia," Review of Financial Studies, 22(11), 4463-4492.

[12] Breeden, Douglas T., and Robert H. Litzenberger, 1978, "Prices of State-Contingent Claims Implicit in Option Prices," Journal of Business, 51(4), 621-651.

[13] Breuer, Peter, 2011 (October), How Does the Volatility Risk Premium Affect the Informational Content of Currency Options? IMF working paper.

[14] Britten-Jones, Mark, and Anthony Neuberger, 2000, "Option Prices, Implied Price Processes, and Stochastic Volatility," Journal of Finance, 55(4), 839-866.

[15] Burgin, Mark, and Gunter Meissner, 2012, "Negative Probabilities in Financial Modeling," Wilmott, 58, 60-65.

[16] Buscha, Thomas, Bent Jesper Christensenb, and Morten rregaard Nielsen, 2011, "The role of implied volatility in forecasting future realized volatility and jumps in foreign exchange, stock, and bond markets," Journal of Econometrics, 160 (1), 48-57.

[17] Campa, Jose M., P.H. Kevin Chang, and Robert L. Reider, 1997 (September), "Implied Exchange Rate Distributions: Evidence from OTC Option Markets," NBER Working Paper No. 6179.

[18] Canina, Linda and Stephen Figlewski, 1993, "The Informational Content of Implied Volatility," Review of Financial Studies, 6 (3), 659-81.

[19] Carr, Peter, and Liuren Wu, 2007, "Stochastic Skew in Currency Options," Journal of Financial Economics, 86(1), 213-247.

[20] Carr, Peter, and Liuren Wu, 2009, "Variance Risk Premiums," Review of Financial Studies, 22(3), 1311-1341.

[21] Castren, Olli, 2004, "Do Options implied Rnd Functions On G3 Currencies Move Around The Times Of Interventions On The Jpy/Usd Exchange Rate?," European Central Bank (ECB) ECB Working Paper No. 410.

[22] Castren, Olli, 2005, "Estimating and Analysing Currency Options Implied Risk-neutral Density Functions for the Largest New EU Member States," European Central Bank (ECB) ECB Working Paper No. 440.

[23] Chen, Yu-chin, and Ranganai Gwati, 2012, "FX Options and Financial Crisis," University of Washington.

[24] Chen, Yu-chin, and Ranganai Gwati, 2013, "FX Options and Excess Returns," University of Washington.

[25] Chiras, Donald P., and Steven Manaster, 1978, "The Information Content of Option Prices and a Test of Market Efficiency," Journal of Financial Economics, 6(2/3), 213-234.

[26] Christensen, Bent J., and Nagpurnanand Prabhala, 1998, "The Relation Between Implied and Realized Volatility," Journal of Financial Economics, 50(2), 125-150.

[27] Christoffersen, Peter F., and Stefano Mazzotta, 2005, "The Accuracy of Density Forecasts from Foreign Exchange Options," Journal of Financial Econometrics, 3(4), 578-605.

[28] Cincibuch, Martin, 2004, "Distributions Implied by American Currency Futures Options: A Ghost's Smile?," Journal of Futures Markets, 24(2), 147-178.

[29] Cooper, Neil, 1999, "Testing techniques for estimating implied RNDs from the prices of European-style Options," Bank of England.

[30] Cooper, Neil, and James Talbot, 1999, "The Yen/Dollar Exchange Rate in 1998: Views from Options Markets. Bank of England Quarterly Bulletin," 39(1), 68-76.

[31] Cox, John, 1975, "Notes on Option Pricing I: Constant Elasticity of Diffusions," Unpublished draft, Stanford University.

[32] Datta, Deepa Dhume, Juan M. Londono, and Landon J. Ross, 2016, "Generating Options-Implied Probability Densities to Understand Oil Market Events" Energy Economics (forthcoming).

[33] Dennis, Patrick, and Stewart Mayhew, 2002, "Risk-Neutral Skewness: Evidence from Stock Options," Journal of Financial and Quantitative Analysis, 37(3), 471-493.

[34] Derman, Emmanual, and Iraj Kani, 1994, "Riding on a Smile," RISK, 7, 32-39.

[35] Dupire, Bruno, 1994, "Pricing with a Smile," Risk, 1, 18-20.

[36] Farhi, Emmanuel, and Xavier Gabaix, 2016, "Rare Disasters and Exchange Rates," Quarterly Journal of Economics, 131(1), 1-52.

[37] Figlewski, Stephen, 2002, "Assessing the Incremental Value of Option Pricing Theory Relative to an Informationally Passive Benchmark," Journal of Derivatives, 10(1), 80-96.

[38] Figlewski, Stephen, 2008, "Estimating the Implied Risk Neutral Density," appeared in Volatility and Time Series Econometrics: Essays in Honor of Robert F. Engle (eds. Tim Bollerslev, Jeffrey R. Russell and Mark Watson), Oxford University Press.

[39] Gabaix, Xavier, Emmanuel Farhi, Samuel Fraiberger, Romain Ranciere, and Adrien Verdelhan, 2016, "Crash Risk in Currency Markets," working paper.

[40] Galai, Dan, and Ben Z. Schreiber, 2013, "Bid-Ask Spreads and Implied Volatility of Key Players in a Fx Options Market," Journal of Futures Markets, 33(8), 774-794.

[41] Gemmill, Gordon, and Apostolos Saflekos, 2000, "How useful are implied distributions? evidence from stock index options," Journal of Derivatives, 7(3), 83-98.

[42] Haug, Espen Gaarder, 2004, "Why so Negative to Negative Probabilities?" Wilmott.

[43] Han, Bing, 2008, "Investor Sentiment and Option Prices," Review of Financial Studies, 21(1), 387-414.

[44] Jarrow, Robert, David Lando, and Fan Yu, 2005, "Default Risk and Diversification: Theory and Empirical Implications," Mathematical Finance, 15(1), 1-26.

[45] Jiang, J. George, and Yisong S. Tian, 2005, "The Model-Free Implied Volatility and Its Information Content," Review of Financial Studies, 18(4), 1305-1342.

[46] John, Hull, 2015, "Options, Futures, and Other Derivatives," Pearson Education, Inc.

[47] Jondeaua, Eric, and Michael Rockingerb, December 2000, "Reading the smile: the message conveyed by methods which infer risk neutral densities," Journal of International Money and Finance, 19(6), 885915.

[48] Jorion, Philippe, 1995, "Predicting Volatility in the Foreign-exchange Market," Journal of Finance, 50(2), 507-528.

[49] Jurek, Jakub W., and Zhikai Xu, 2014, "Option-Implied Currency Risk Premia," working paper.

[50] Khrapov, Stanislav September 15, 2014, "Option Pricing via Risk-Neutral Density Forecasting," working paper.

[51] Kitsul, Yuriy, and Jonathan H. Wright, 2013, "The economics of options-implied inflation probability density functions," Journal of Financial Economics, 110 (3), 696-711.

[52] Kozhan, Roman, Anthony Neuberger, and Paul Schneider, 2013, "The Skew Risk Premium in the Equity Index Market," Review of Financial Studies, 26(9), 2174-2203.

[53] Liu, Jun, Jun Pan, and Tan Wang, 2005, "An Equilibrium Model of Rare-Event Premia and Its Implication for Option Smirks," Review of Financial Studies, 18(1), 131-164.

[54] Malz, Allan, 1997, "Estimating the Probability Distribution of the Future Exchange Rate from Option Prices," The Journal of Derivatives, 5(2), 18-36.

[55] Malz, Allan, 1997 (November), "Option-implied Probability Distributions and Currency Excess Returns," New York FED #32.

[56] Malz, Allan, 2014 (June), "A Simple and Reliable Way to Compute Option-Based Risk-Neutral Distributions," New York FED #677.

[57] Nitteberg, Morten Bergendahl, 2011, "Implied Risk-Neutral Densities: An application to the WTI Crude Oil market," working paper.

[58] Orosi, Greg, 2015, "Estimating Option-implied Risk-neutral Densities: A Novel Parametric Approach," The Journal of Derivatives, 23(1), 41-61.

[59] Rubinstein, Mark, 1994, "Implied Binomial Trees," Journal of Finance, 49(3), 771-818.

[60] Santos, Andr, and Joo Guerra, 2015 (July), "Implied Risk Neutral Densities From Option Prices: Hypergeometric, Spline, Lognormal, and Edgeworth Functions," Journal of Futures Markets, 35(7), 655 678.

[61] Shimko, David, 1993 (April), "Bounds of Probability," Risk, 6(4), 33-37.

[62] Xu, XinZhong, and Stephen Taylor, 1994, "The Term Structure of Volatility Implied by Foreign Exchange Options," Journal of Financial and Quantitative Analysis, 29(1), 57-74.

[63] Zhou, Yinggang, Yi-Ming Wang, and Xin Cheng, 2016, "Why Is RMB increasingly important? A Network Approach," Xiamen University.

5 Appendix

5.1 Piece-wise Linear RND

The function is demonstrated in Panel (a) of Figure 8. We write the density function for any strike $k = 0, \cdots, n$ as follows:

$$g_k(S) = \frac{a_k - a_{k-1}}{K_{k+1} - K_k}(S - K_k) + a_{k-1} \tag{20}$$

where $a_{-1} = 0$, $K_0 = 0$, and $K_{n+1} = x$ for. The density function needs to integrate to 1:

$$\begin{aligned} 1 &= \int_0^{K_1} \frac{a_0}{K_1} S dS + \int_{K_1}^{K_2} \left\{ \frac{a_1 - a_0}{K_2 - K_1} S + \frac{a_0 K_2 - a_1 K_1}{K_2 - K_1} \right\} dS + \\ &\cdots + \int_{K_{n-1}}^{K_n} \left\{ \frac{a_{n-1} - a_{n-2}}{K_n - K_{n-1}} S + \frac{a_{n-2} K_n - a_n K_n}{K_n - K_{n-1}} \right\} dS \\ &= \sum\nolimits_{k=0}^{n} \frac{1}{2}(K_{k+1} - K_k)(a_k + a_{k-1}) \end{aligned}$$

where $j \geqslant k$ for the option to have a positive payoff. This equation is used to solve for $x = K_{n+1}$ as follows:

$$x = \frac{2}{a_n + a_{n-1}} \left(1 - \sum\nolimits_{k=1}^{n} \frac{1}{2}(K_{k+1} - K_k)(a_k + a_{k-1}) \right) + K_n \tag{21}$$

The pricing equation of a call option can be derived easily as follows:

$$C_k = \sum\nolimits_{j=k}^{n} v_k(j) \tag{22}$$

where

$$
\begin{aligned}
v_k(j) &= \int_{K_j}^{K_{j+1}} (S-K_k) g_k(S) dS \\
&= \int_{K_j}^{K_{j+1}} (S-K_k) \left[\frac{a_k - a_{k-1}}{K_{k+1}-K_k} S + \frac{a_{k-1}K_{k+1} - a_k K_k}{K_{k+1}-K_k} \right] dS \\
&= \int_{K_j}^{K_{j+1}} \left(\alpha_k S^2 + \beta_k S + \gamma_k \right) dS \\
&= \left\{ \frac{\alpha_k}{3} S^3 + \frac{\beta_k}{2} S^2 + \gamma_k S \right\} \Bigg|_{K_j}^{K_{j+1}}
\end{aligned} \tag{23}
$$

and

$$
\begin{aligned}
\alpha_k &= \frac{a_k - a_{k-1}}{K_{k+1}-K_k} \\
\beta_k &= -K_k \frac{a_k - a_{k-1}}{K_{k+1}-K_k} + \frac{a_{k-1}K_{k+1} - a_k K_k}{K_{k+1}-K_k} \\
\gamma_k &= -K_k \frac{a_{k-1}K_{k+1} - a_k K_k}{K_{k+1}-K_k}
\end{aligned}
$$

and hence $v_k(j)$ can be easily computed.

The moments of the RND of the underlying asset are shown as follows:

$$
\begin{aligned}
\mathbb{E}[S^m] &= \sum\nolimits_{k=0}^{n} \int_{K_k}^{K_{k+1}} S^m g_k(S) dS \\
&= \sum\nolimits_{k=0}^{n} \int_{K_k}^{K_{k+1}} S^m \left[\frac{a_k - a_{k-1}}{K_{k+1}-K_k} S + \frac{a_{k-1}K_{k+1} - a_k K_k}{K_{k+1}-K_k} \right] dS \\
&= \sum\nolimits_{k=0}^{n} \int_{K_k}^{K_{k+1}} \left(\alpha_k S^{m+1} + c_k S^m \right) dS \\
&= \sum\nolimits_{k=0}^{n} \left\{ \frac{\alpha_k}{m+2} S^{m+2} + \frac{c_k}{m+1} S^{m+1} \right\} \Bigg|_{K_k}^{K_{k+1}}
\end{aligned} \tag{24}
$$

where $a_{-1} = 0$ and

$$
\begin{aligned}
\alpha_k &= \frac{a_k - a_{k-1}}{K_{k+1}-K_k} \\
c_k &= \frac{a_{k-1}K_{k+1} - a_k K_k}{K_{k+1}-K_k}
\end{aligned}
$$

When $m = 0$ it is the integration of the p.d.f. which is 1. When $m > 0$, it is an uncentralized moment.

5.2 Cubic-spline RND

The function is presented graphically in Panel (b) of Figure 8. The equation for the function is:

$$g_k(S) = \begin{cases} a_k S^3 + b_k S^2 + c_k S + d_k & k = 2, \cdots, 7 \\ c_k S + d_k & k = 1, 8 \end{cases} \tag{25}$$

where the following constants are satisfied (for $k = 2, \cdots, 7$):

$$\begin{aligned} g_{k-1}(K_k) &= g_k(K_k) \\ g'_{k-1}(K_k) &= g'_k(K_k) \\ g''_{k-1}(K_k) &= g''_k(K_k) \end{aligned}$$

to guarantee smoothness (twice differentiable). There are a total of six strikes: 3 calls and 3 puts (ATM, 25-delta, and 10-delta – ATM call and and ATM put do not have the same strike). These are $k = 2, \cdots, 7$. K_1 and K_8 are two hypothetical strike values whose corresponding option prices are computed using the same volatility quotes of K_2 and K_7. The upper (U) and lower (L) limits of the RND are obtained to guarantee the probability to be 1.

5.3 Piece-wise Log Linear RND

We rewrite (20) as:

$$g_k(\ln S) = \frac{b_k - b_{k-1}}{\ln K_{k+1} - \ln K_k}(x - \ln K_k) + b_{k-1}$$

and (21) as:

$$x = \frac{2}{b_n + b_{n-1}} \left(1 - \sum\nolimits_{k=1}^{n} \frac{1}{2} (\ln K_{k+1} - \ln K_k)(b_k + b_{k-1})\right) + \ln K_n \tag{26}$$

The pricing equation for the call option remains as (22) with the following replacement for (23):

$$\begin{aligned} v_k(j) &= \int_{\ln K_j}^{\ln K_{j+1}} (e^x - e^{\ln K_k}) g_k(x) dx \\ &\approx \int_{\ln K_j}^{\ln K_{j+1}} (x - \ln K_k) g_k(x) dx \end{aligned} \tag{27}$$

In short, it is the same solution except that S is replaced by $\ln S$ and K is replaced by $\ln K$.

Table 5: Regression over Net Imports to US from EU

(a) 4Ms Regression on Net Imports

coef												
Depedent_Var	Cnst	**Spread**	**D_net_lag3**	**D_net_lag2**	**D_net_lag1**	**D_net**	**D_net_Lead1**	**D_net_Lead2**	**D_net_Lead3**	**D_net_Lead4**	**D_net_Lead5**	**D_net_Lead6**
1W 4M	-9.86E-06	0.00013	-2.88E-08	-1.19E-09	3.95E-09	1.63E-08	7.32E-10	3.04E-08	3.31E-08	3.96E-08	2.14E-08	1.74E-09
2W 4M	-5.47E-06	0.0000981	-3.21E-08	9.04E-09	1.58E-08	2.49E-08	1.91E-08	4.83E-08	5.45E-08	5.71E-08	2.84E-08	-6.43E-09
1M 4M	0.0000106	0.0000958	-5.15E-08	1.73E-08	3.16E-08	5.26E-08	6.58E-08	9.71E-08	1.05E-07	1.03E-07	6.90E-08	1.34E-08
2M 4M	0.0000455	0.000105	-7.28E-08	3.03E-08	7.43E-08	1.26E-07	1.47E-07	1.89E-07	1.73E-07	1.67E-07	1.23E-07	4.99E-08
3M 4M	0.000101	0.000102	-7.03E-08	6.74E-08	1.36E-07	2.05E-07	2.45E-07	2.99E-07	2.62E-07	2.25E-07	1.82E-07	7.59E-08
6M 4M	0.000265	0.000114	-6.91E-08	1.76E-07	3.28E-07	4.87E-07	5.09E-07	5.81E-07	4.82E-07	4.07E-07	3.33E-07	1.67E-07
9M 4M	0.000449	0.000123	-6.93E-08	3.31E-07	5.66E-07	8.12E-07	7.96E-07	8.62E-07	7.17E-07	5.94E-07	5.25E-07	2.43E-07
1Y 4m	0.00066	0.000129	-7.86E-08	4.90E-07	8.35E-07	1.20E-06	1.12E-06	1.18E-06	9.85E-07	7.93E-07	7.36E-07	3.44E-07
t stat												
Depedent_Var	Cnst	**Spread**	**D_net_lag3**	**D_net_lag2**	**D_net_lag1**	**D_net**	**D_net_Lead1**	**D_net_Lead2**	**D_net_Lead3**	**D_net_Lead4**	**D_net_Lead5**	**D_net_Lead6**
1W 4M	-0.62	5.82	-2.28	-0.08	0.24	0.99	0.04	**1.86**	**2.01**	2.34	1.4	0.14
2W 4M	-0.27	6.5	-2.06	0.45	0.73	1.14	0.87	**2.21**	**2.47**	2.58	1.38	-0.41
1M 4M	0.34	8.5	-2.12	0.54	0.93	1.54	1.9	**2.82**	**3.03**	2.91	2.1	0.54
2M 4M	0.97	10.24	-2.06	0.63	1.46	2.46	2.83	**3.66**	**3.3**	3.15	2.5	1.38
3M 4M	1.7	11.03	-1.52	1.08	2.06	3.12	3.63	**4.48**	**3.86**	3.28	2.81	1.59
6M 4M	2.68	13.14	-0.88	1.69	2.99	4.45	4.5	**5.21**	**4.23**	3.57	3.08	2.08
9M 4M	3.11	14.28	-0.61	2.25	3.63	5.15	4.89	**5.37**	**4.37**	3.68	3.46	2.11
1Y 4m	3.4	14.64	-0.51	2.47	3.98	5.62	5.09	**5.47**	**4.45**	3.64	3.6	2.2

(b) Spreads Regression on Net Imports

coef												
Depedent_Var	Cnst	**4M**	**D_net_lag3**	**D_net_lag2**	**D_net_lag1**	**D_net**	**D_net_Lead1**	**D_net_Lead2**	**D_net_Lead3**	**D_net_Lead4**	**D_net_Lead5**	**D_net_Lead6**
1W spread	0.0931	1557	0.0000636	0.000105	0.000109	-0.000021	-5.30E-07	-0.000042	-0.000029	0.0000359	0.000049	0.0000311
2W spread	0.1476	3116	0.000094	0.000119	0.000106	-0.000064	-0.000018	-0.000114	-0.000137	-0.000058	0.0000116	0.0000585
1M spread	0.2285	4393	0.000212	0.000176	0.000129	-0.000186	-0.000166	-0.000309	-0.000304	-0.000179	-0.000057	0.0000352
2M spread	0.3061	4777	0.000414	0.000359	0.0000927	-0.000463	-0.000375	-0.000625	-0.000394	-0.000335	-0.000157	-0.000175
3M spread	0.2576	5130	0.000446	0.000339	-0.000013	-0.000732	-0.00066	-0.001053	-0.000653	-0.000535	-0.000321	-0.000302
6M spread	-0.0197	5275	0.000707	0.000192	-0.000576	-0.001679	-0.001381	-0.002054	-0.001255	-0.001224	-0.000806	-0.00071
9M spread	-0.2084	4998	0.001011	-0.000102	-0.001186	-0.00251	-0.00194	-0.002871	-0.001848	-0.001813	-0.001126	-0.000812
1Y spread	-0.4874	4793	0.001263	-0.000455	-0.001868	-0.003466	-0.002644	-0.003872	-0.002571	-0.00242	-0.001562	-0.00096
t stat												
Depedent_Var	Cnst	**4M**	**D_net_lag3**	**D_net_lag2**	**D_net_lag1**	**D_net**	**D_net_Lead1**	**D_net_Lead2**	**D_net_Lead3**	**D_net_Lead4**	**D_net_Lead5**	**D_net_Lead6**
1W spread	1.62	3.78	1.18	1.63	1.64	-0.32	-0.01	-0.66	-0.44	0.52	0.73	0.59
2W spread	1.14	5.93	1.01	1.02	0.83	-0.52	-0.15	-0.92	-1.08	-0.44	0.1	0.63
1M spread	0.9	8.12	1.22	0.77	0.51	-0.76	-0.69	-1.27	-1.22	-0.69	-0.24	0.2
2M spread	0.74	10.16	1.7	1	0.23	-1.19	-0.97	-1.6	-0.99	-0.82	-0.42	-0.7
3M spread	0.47	10.16	1.31	0.68	-0.02	-1.38	-1.24	-1.95	-1.18	-0.96	-0.62	-0.86
6M spread	-0.02	12.08	1.32	0.24	-0.66	-1.96	-1.6	-2.36	-1.42	-1.39	-0.99	-1.29
9M spread	-0.17	14.16	1.46	-0.1	-1.05	-2.25	-1.73	-2.54	-1.61	-1.63	-1.09	-1.16
1Y spread	-0.31	14.96	1.43	-0.34	-1.33	-2.47	-1.88	-2.74	-1.79	-1.75	-1.21	-1.07

Table 6: FX Prediction

delta 1W_Spot (#1838)	Coef	t-stat	Coef	t-stat	delta 2W_Spot (#1833)	Coef	t-stat	Coef	t-stat
Const	0.00	-0.88	0.00	-1.01	Const	0.00	-0.63	0.00	-0.77
d_1W_M2	-156.32	-6.38			d_2W_M2	-12.03	-0.56		
d_1W_M3	-371.08	-6.82			d_2W_M3	-426.76	-11.69		
d_1W_M4	-187.58	-6.86			d_2W_M4	-376.07	-11.55		
d_1W_IV	237.23	6.13	21.17	2.57	d_2W_IV	40.66	1.40	20.27	3.37
Lag1_delta_1W_spot	0.89	29.94	0.88	29.20	Lag1_delta_2W_spot	0.94	37.75	0.97	33.30
Lag2_delta_1W_spot	0.02	0.44	-0.01	-0.13	Lag2_delta_2W_spot	0.05	1.54	-0.03	-0.65
Lag3_delta_1W_spot	-0.04	-0.96	-0.03	-0.74	Lag3_delta_2W_spot	-0.03	-1.04	-0.06	-1.48
Lag4_delta_1W_spot	-0.05	-2.01	-0.09	-2.95	Lag4_delta_2W_spot	0.01	0.42	0.01	0.23
adj_R2	69.85%		66.64%		adj_R2	87.40%		82.35%	
delta 1M_Spot (#1822)	**Coef**	**t-stat**	**Coef**	**t-stat**	**delta 2M_Spot (#1802)**	**Coef**	**t-stat**	**Coef**	**t-stat**
Const	0.00	-0.69	0.00	-0.71	Const	0.00	-0.86	0.00	-0.80
d_1M_M2	12.84	3.15			d_2M_M2	2.20	0.71		
d_1M_M3	-208.01	-9.66			d_2M_M3	-90.89	-9.93		
d_1M_M4	-197.00	-11.05			d_2M_M4	-83.30	-9.05		
d_1M_IV	9.71	2.00	15.47	3.66	d_2M_IV	20.00	4.78	10.19	3.82
Lag1_delta_1M_spot	0.95	37.38	1.01	33.69	Lag1_delta_2M_spot	0.93	39.28	1.00	32.84
Lag2_delta_1M_spot	0.02	0.64	-0.03	-0.63	Lag2_delta_2M_spot	0.08	2.19	0.02	0.47
Lag3_delta_1M_spot	0.03	0.74	-0.02	-0.66	Lag3_delta_2M_spot	-0.04	-1.16	-0.07	-1.71
Lag4_delta_1M_spot	-0.02	-0.71	0.00	-0.08	Lag4_delta_2M_spot	0.02	0.69	0.03	0.90
adj_R2	93.67%		91.89%		adj_R2	96.78%		95.97%	
delta 3M_Spot (#1781)	**Coef**	**t-stat**	**Coef**	**t-stat**	**delta 6M_Spot (#1718)**	**Coef**	**t-stat**	**Coef**	**t-stat**
Const	0.00	-1.06	0.00	-1.01	Const	0.00	-1.01	0.00	-1.02
d_3M_M2	-9.69	-2.56			d_6M_M2	-21.22	-11.81		
d_3M_M3	-39.27	-3.07			d_6M_M3	-12.38	-4.85		
d_3M_M4	-28.19	-2.24			d_6M_M4	7.49	2.68		
d_3M_IV	31.59	6.65	8.33	4.65	d_6M_IV	37.66	12.46	5.31	3.25
Lag1_delta_3M_spot	0.95	40.38	0.98	37.21	Lag1_delta_6M_spot	0.95	41.55	1.00	36.92
Lag2_delta_3M_spot	0.04	1.20	0.02	0.52	Lag2_delta_6M_spot	0.04	1.30	0.00	-0.07
Lag3_delta_3M_spot	-0.03	-0.81	-0.05	-1.24	Lag3_delta_6M_spot	-0.04	-1.09	-0.04	-0.98
Lag4_delta_3M_spot	0.03	1.29	0.04	1.31	Lag4_delta_6M_spot	0.04	1.70	0.04	1.50
adj_R2	97.77%		97.35%		adj_R2	99.04%		98.69%	
delta 9M_Spot (#1655)	**Coef**	**t-stat**	**Coef**	**t-stat**	**delta 1Y_Spot (#1593)**	**Coef**	**t-stat**	**Coef**	**t-stat**
Const	0.00	-0.82	0.00	-0.73	Const	0.00	-0.95	0.00	-0.75
d_9M_M2	-16.15	-15.02			d_1Y_M2	-13.00	-16.61		
d_9M_M3	-8.73	-5.11			d_1Y_M3	-5.75	-7.46		
d_9M_M4	5.60	3.09			d_1Y_M4	5.50	5.46		
d_9M_IV	29.70	17.45	5.08	4.79	d_1Y_IV	23.39	20.35	4.13	4.68
Lag1_delta_9M_spot	0.96	46.15	1.00	39.74	Lag1_delta_1Y_spot	0.95	46.23	0.99	33.68
Lag2_delta_9M_spot	0.05	1.53	-0.01	-0.18	Lag2_delta_1Y_spot	0.07	2.28	0.03	0.84
Lag3_delta_9M_spot	-0.01	-0.50	0.00	-0.06	Lag3_delta_1Y_spot	-0.03	-1.02	-0.03	-0.70
Lag4_delta_9M_spot	0.01	0.21	0.01	0.23	Lag4_delta_1Y_spot	0.01	0.44	0.01	0.24
adj_R2	99.31%		98.99%		adj_R2	99.36%		98.98%	

Table 7: Granger-Causality Wald Test for EPU and RND (Lag=6)

(a) Term Structure of Moments v.s. EPU

Test	Chi-Square	Pr >ChiSq	ReMark
Term Struc. IV doesn't predict EPU	135.59	<0.0001	Reject
EPU doesn't predict Term Struc. IV	103.49	<0.0001	Reject
Term Struc. M2 doesn't predict EPU	113.15	<0.0001	Reject
EPU doesn't predict Term Struc. M2	82.73	0.0014	Doesn't Reject
Term Struc. M3 doesn't predict EPU	102.07	<0.0001	Reject
EPU doesn't predict Term Struc. M3	64.11	0.0598	Doesn't Reject
Term Struc. M4 doesn't predict EPU	92.24	<0.0001	Reject
EPU doesn't predict Term Struc. M4	66.49	0.0397	Doesn't Reject

(b) Moments v.s. EPU

Test	Chi-Square	Pr>ChiSq	ReMark
1-Week Moments doesn't predict EPU	57.98	<0.0001	Reject
EPU doesn't predict 1-Week Moments	28.88	0.0498	Doesn't Reject
2-Week Moments doesn't predict EPU	58.31	<0.0001	Reject
EPU doesn't predict Term Struc. 2-Week	34.2	0.0119	Doesn't Reject
1-Month Moments doesn't predict EPU	53.78	<0.0001	Reject
EPU doesn't predict 1-Month Moments	38.71	0.0031	Doesn't Reject
2-Month Moments doesn't predict EPU	49.17	<0.0001	Reject
EPU doesn't predict 2-Month Moments	34.39	0.0113	Doesn't Reject
3-Month Moments doesn't predict EPU	43.91	0.0006	Doesn't Reject
EPU doesn't predict 3-Month Moments	26.79	0.083	Doesn't Reject
6-Month Moments doesn't predict EPU	38.92	0.0029	Doesn't Reject
EPU doesn't predict 6-Month Moments	26.55	0.0879	Doesn't Reject
9-Month Moments doesn't predict EPU	37.94	0.0039	Doesn't Reject
EPU doesn't predict 9-Month Moments	33.18	0.0159	Doesn't Reject
1-Year Moments doesn't predict EPU	40.82	0.0016	Doesn't Reject
EPU doesn't predict 1-Year Moments	28.75	0.0515	Doesn't Reject

Table 8: Granger-Causality Wald Test for USD Influence Index and RND (Lag=1)

(a) Term Structure v.s. USD Influence Index

Test	Chi-Square	Pr >ChiSq	ReMark
Term Struc. IV doesn't predict Influence Index	108.79	<0.0001	Reject
Influence Index doesn't predict Term Struc. IV	73.59	<0.0001	Reject
Term Struc. M2 doesn't predict Influence Index	95.96	<0.0001	Reject
Influence Index doesn't predict Term Struc. M2	7.92	0.4416	Doesn't Reject
Term Struc. M3 doesn't predict Influence Index	68.11	<0.0001	Reject
Influence Index doesn't predict Term Struc. M3	15.77	0.04586	Doesn't Reject
Term Struc. M4 doesn't predict Influence Index	72.86	<0.0001	Reject
Influence Index doesn't predict Term Struc. M4	16.11	0.0409	Doesn't Reject

(b) Moments v.s. USD Influence Index

Test	Chi-Square	Pr>ChiSq	ReMark
1-Week Moments doesn't predict Influence Index	89.75	<0.0001	Reject
Influence Index doesn't predict 1-Week Moments	5.37	0.1469	Doesn't Reject
2-Week Moments doesn't predict Influence Index	77.4	<0.0001	Reject
Influence Index doesn't predict 2-Week Moments	8.69	0.0337	Doesn't Reject
1-Month Moments doesn't predict Influence Index	75.49	<0.0001	Reject
Influence Index doesn't predict 1-Month Moments	5.92	0.1153	Doesn't Reject
2-Month Moments doesn't predict Influence Index	76.87	<0.0001	Reject
Influence Index doesn't predict 2-Month Moments	10.48	0.0149	Doesn't Reject
3-Month Moments doesn't predict Influence Index	71.9	<0.0001	Reject
Influence Index doesn't predict 3-Month Moments	13.48	0.0037	Doesn't Reject
6-Month Moments doesn't predict Influence Index	68.43	<0.0001	Reject
Influence Index doesn't predict 6-Month Moments	18.51	0.0003	Doesn't Reject
9-Month Moments doesn't predict Influence Index	66.13	<0.0001	Reject
Influence Index doesn't predict 9-Month Moments	18.32	0.0004	Doesn't Reject
1-Year Moments doesn't predict Influence Index	129.21	<0.0001	Reject
Influence Index doesn't predict 1-Year Moments	31.3	0.0018	Doesn't Reject

Table 9: Future Realized Volatility Explanation Ability

RZ_VAR 1W	Coef	t stat
Cnst	0	2.85
M2	0.53	2.1
M3	1.32	2.72
M4	0.72	3.12
IV	-0.22	-0.94
#	1843	
adj. R2	17.56%	

RZ_VAR 2W	Coef	t stat
Cnst	0	5.09
M2	0.05	0.22
M3	0.67	1.57
M4	0.51	2.37
IV	0.21	0.98
#	1838	
adj. R2	15.79%	

RZ_VAR 1M	Coef	t stat
Cnst	0	8.94
M2	-0.02	-0.16
M3	0.3	1.38
M4	0.11	0.75
IV	0.35	3.28
#	1822	
adj. R2	20.70%	

RZ_VAR 2M	Coef	t stat
Cnst	0	12.89
M2	-0.18	-1.73
M3	-0.43	-2.82
M4	-0.58	-4.82
IV	0.5	5.33
#	1801	
adj. R2	29.86%	

RZ_VAR 3M	Coef	t stat
Cnst	0	13.78
M2	-0.12	-1.22
M3	-0.5	-4.43
M4	-0.71	-7.03
IV	0.4	4.44
#	1781	
adj. R2	34.14%	

RZ_VAR 6M	Coef	t stat
Cnst	0	7.1
M2	0.68	8.02
M3	-0.36	-5.32
M4	-1.22	-15.46
IV	-0.29	-3.81
#	1718	
adj. R2	54.48%	

RZ_VAR 9M	Coef	t stat
Cnst	0	10.02
M2	0.79	10.8
M3	0.19	4.24
M4	-0.79	-12.64
IV	-0.51	-8.18
#	1655	
adj. R2	61.96%	

RZ_VAR 1Y	Coef	t stat
Cnst	0.01	13.87
M2	0.47	7.17
M3	0.43	13.24
M4	-0.25	-4.71
IV	-0.39	-7.28
#	1592	
adj. R2	54.93%	

Table 10: Regression: (a) VIX on Concurrent Moments

1W		
	Coef	t stat
Cnst	11.01	23.71
M2	8964.07	3.66
M3	16508.02	3.83
M4	8746.57	4.07
IV	22523.54	5.70
#	1799	
adj. R2	73.20%	

2W		
	Coef	t stat
Cnst	10.80	27.39
M2	-2553.36	-1.19
M3	7085.26	3.63
M4	9413.07	6.12
IV	24459.10	7.23
#	1799	
adj. R2	77.30%	

1M		
	Coef	t stat
Cnst	10.63	32.25
M2	122.44	0.18
M3	1831.58	2.10
M4	1529.04	1.50
IV	10410.12	9.63
#	1799	
adj. R2	77.60%	

2M		
	Coef	t stat
Cnst	9.56	24.76
M2	513.60	1.25
M3	-880.43	-1.86
M4	-1568.25	-2.66
IV	5360.69	8.47
#	1799	
adj. R2	76.85%	

3M		
	Coef	t stat
Cnst	9.01	21.68
M2	93.55	0.33
M3	-400.78	-1.39
M4	-778.60	-2.15
IV	4185.75	9.50
#	1799	
adj. R2	75.20%	

6M		
	Coef	t stat
Cnst	10.50	23.65
M2	-1058.48	-7.51
M3	723.56	6.31
M4	1091.14	7.30
IV	3697.50	17.24
#	1799	
adj. R2	71.88%	

9M		
	Coef	t stat
Cnst	11.14	23.41
M2	-913.49	-9.45
M3	673.00	10.85
M4	1002.31	11.30
IV	2627.02	18.24
#	1799	
adj. R2	69.69%	

1Y		
	Coef	t stat
Cnst	11.64	22.77
M2	-716.02	-9.64
M3	535.22	13.59
M4	788.01	12.86
IV	1901.57	17.59
#	1799	
adj. R2	67.50%	

Note: The regression is VIX(t) on Moments(t-22).

Regression: (b) VIX on Lagged Moments

1W

	Coef	t stat
Cnst	10.64	19.43
M2	27696.20	9.45
M3	51089.20	9.92
M4	25049.13	9.81
IV	-11473.93	-2.42
#	1777	
adj. R2	63.58%	

2W

	Coef	t stat
Cnst	11.72	23.27
M2	5932.24	2.15
M3	26946.55	10.67
M4	20156.26	10.18
IV	8641.78	1.99
#	1777	
adj. R2	63.77%	

1M

	Coef	t stat
Cnst	13.78	31.02
M2	-594.88	-0.65
M3	11565.47	9.65
M4	10437.76	7.50
IV	9744.04	6.68
#	1777	
adj. R2	59.84%	

2M

	Coef	t stat
Cnst	14.34	27.32
M2	-817.81	-1.46
M3	3859.47	5.93
M4	3197.28	3.97
IV	6175.66	7.14
#	1777	
adj. R2	57.74%	

3M

	Coef	t stat
Cnst	14.16	25.50
M2	-996.85	-2.59
M3	2093.23	5.42
M4	1880.91	3.89
IV	4907.07	8.27
#	1777	
adj. R2	56.33%	

6M

	Coef	t stat
Cnst	15.86	28.70
M2	-1850.94	-10.51
M3	1416.19	9.91
M4	2059.17	11.10
IV	4368.07	16.29
#	1777	
adj. R2	57.09%	

9M

	Coef	t stat
Cnst	16.40	28.81
M2	-1513.21	-13.06
M3	945.54	12.73
M4	1518.98	14.34
IV	3170.67	18.37
#	1777	
adj. R2	57.26%	

1Y

	Coef	t stat
Cnst	16.72	27.93
M2	-1155.86	-13.28
M3	659.67	14.27
M4	1095.51	15.28
IV	2288.71	18.05
#	1777	
adj. R2	55.97%	

Note: The regression is VIX(t) on Moments(t-22).

Table 11: Estimated Heston Parameters

	kappa		theta		gamma	
	coef.	std.err.	coef.	std.err.	coef.	std.err.
1W	22.9153	0.0117	0.0076	0.0007	0.0186	0
2W	7.3978	0.0067	0.0110	0.0003	0.0151	0
1M	5.2884	0.0057	0.0174	0.0004	0.0226	0
2M	3.9531	0.0049	0.0267	0.0004	0.0334	0
3M	3.2716	0.0045	0.035	0.0005	0.0434	0
6M	2.6491	0.0041	0.0548	0.0006	0.0757	0
9M	2.2670	0.0038	0.0701	0.0006	0.1077	0
1Y	2.0049	0.0036	0.0831	0.0007	0.1413	0.0001

Table 12: Estimated CEV Parameters

	beta	kappa	theta	gamma
mean	1.8816	1.0071	0.1979	0.2574
median	1.9897	0.9691	0.1917	0.2655
std.dev.	0.3849	0.3483	0.131	0.1421

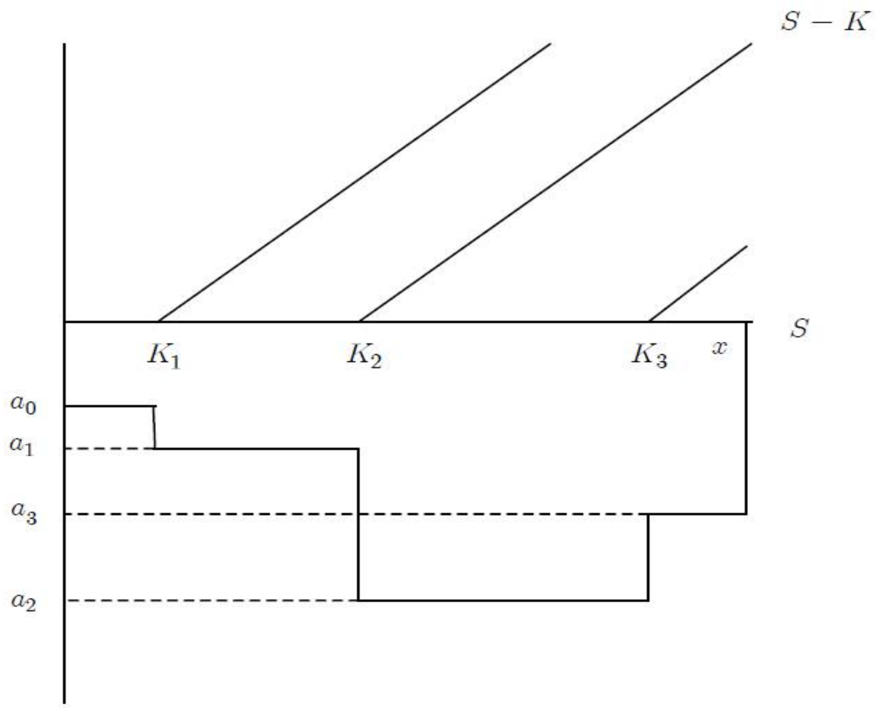
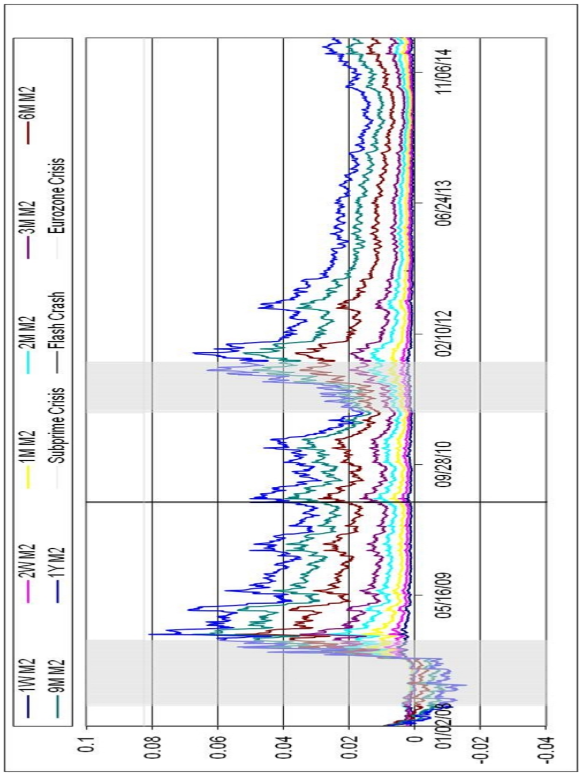
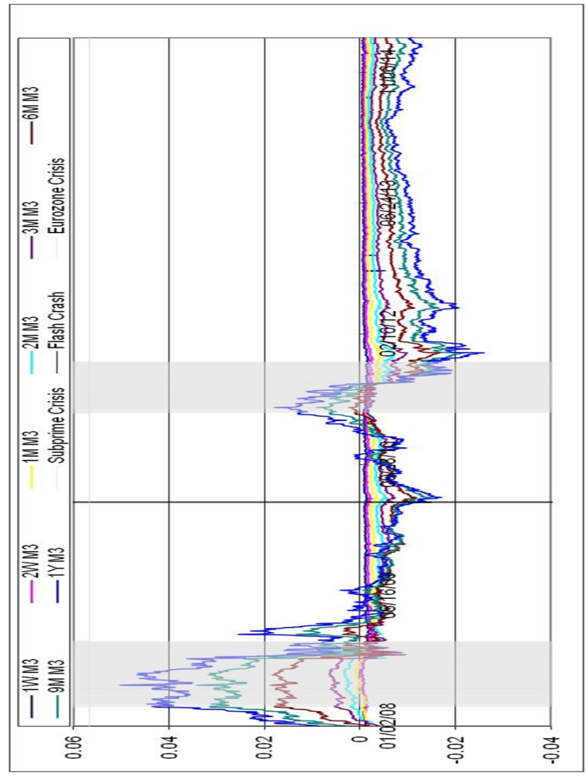


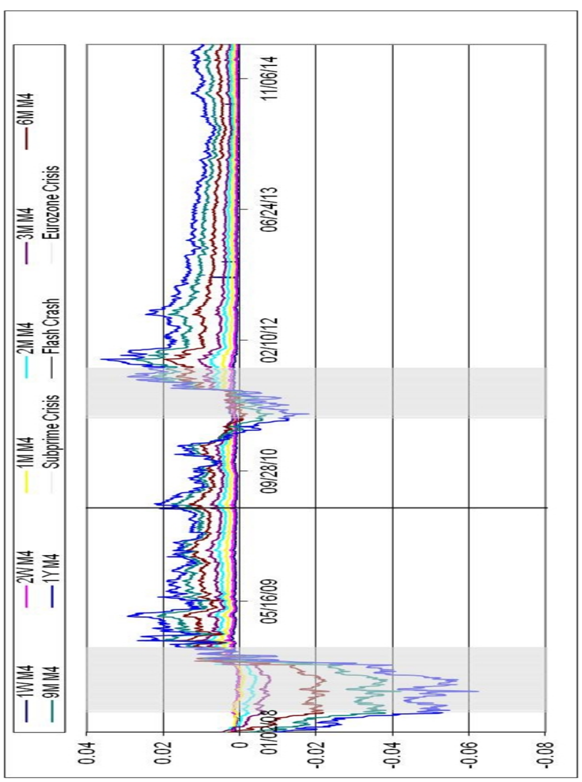
Figure 1: Piece-wise Flat RND



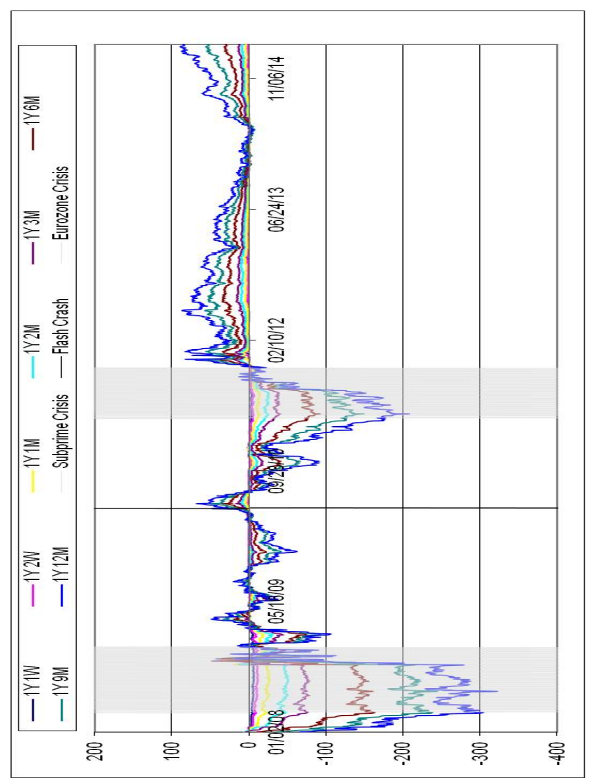
(a) Variance



(b) Skewness



(c) Kurtosis



(d) SWAP Spread

Figure 2: Moments of RND

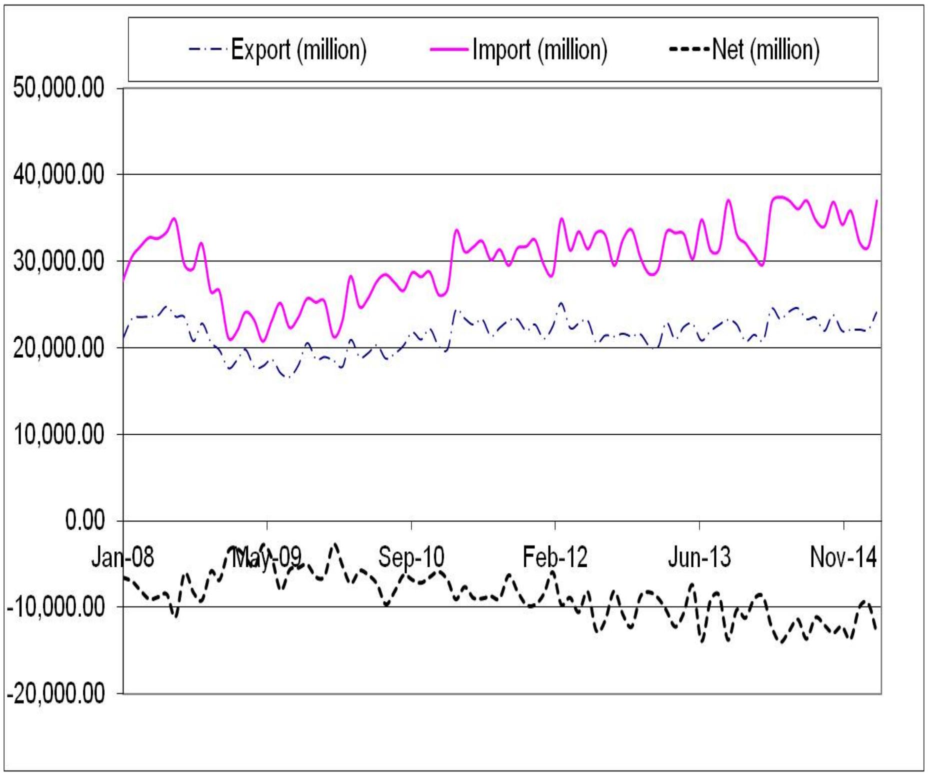
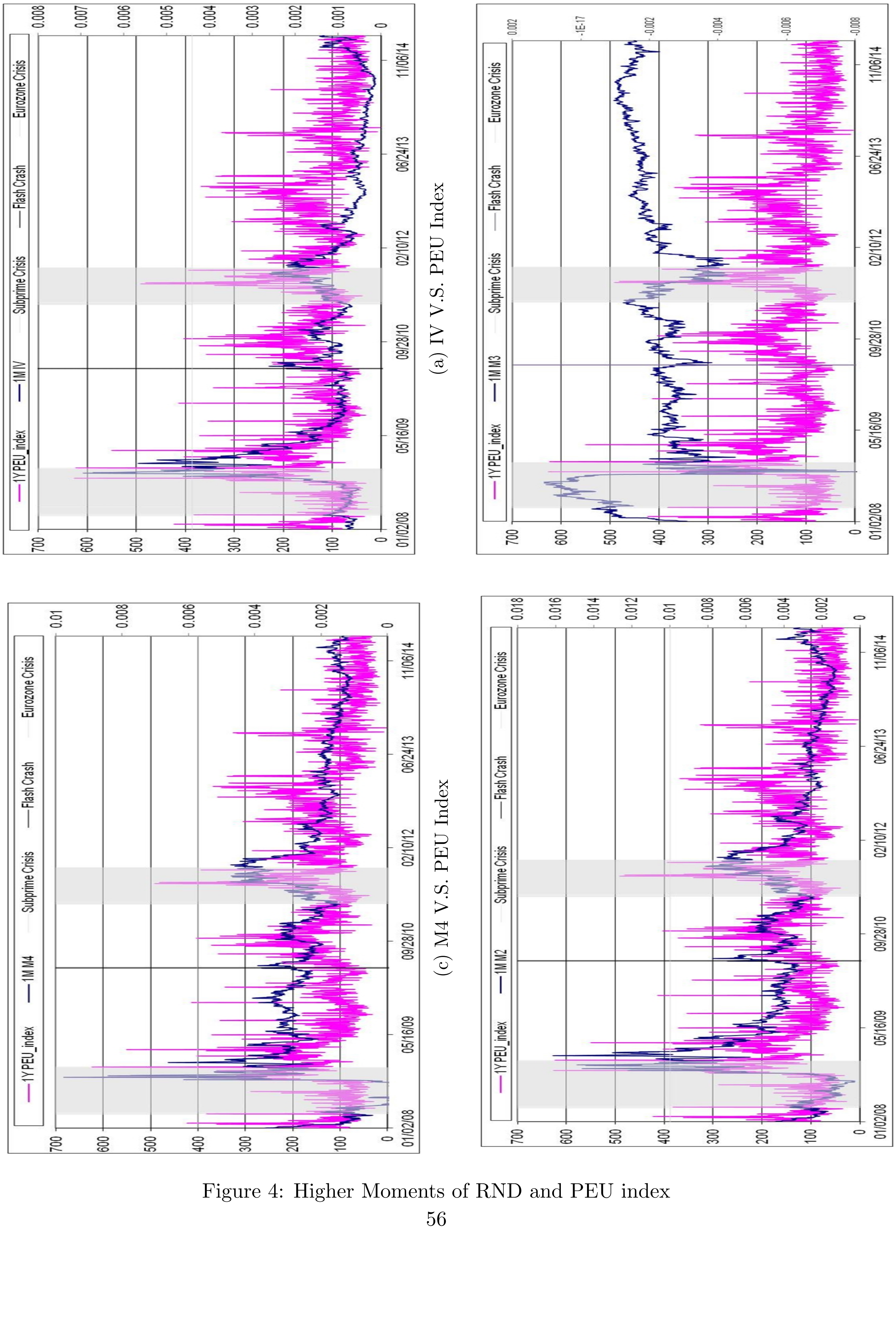


Figure 3: Net Import to US From EU



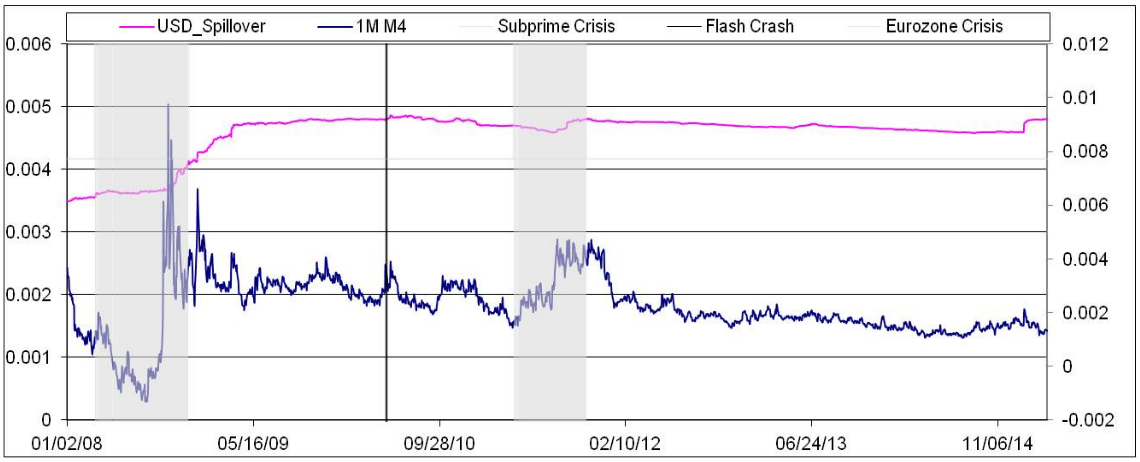
(a) IV V.S. PEU Index

(b) M3 V.S. PEU Index

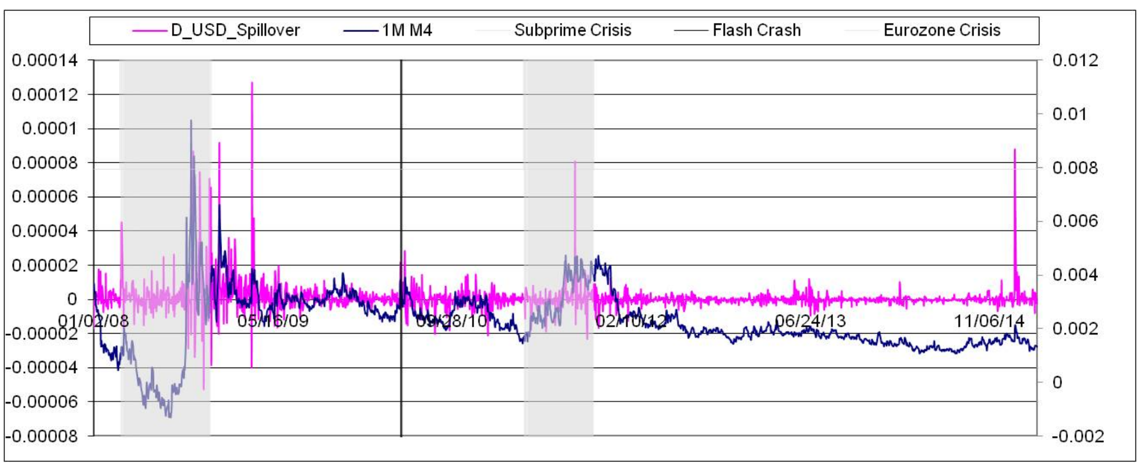
(c) M4 V.S. PEU Index

(d) M2 V.S. PEU Index

Figure 4: Higher Moments of RND and PEU index

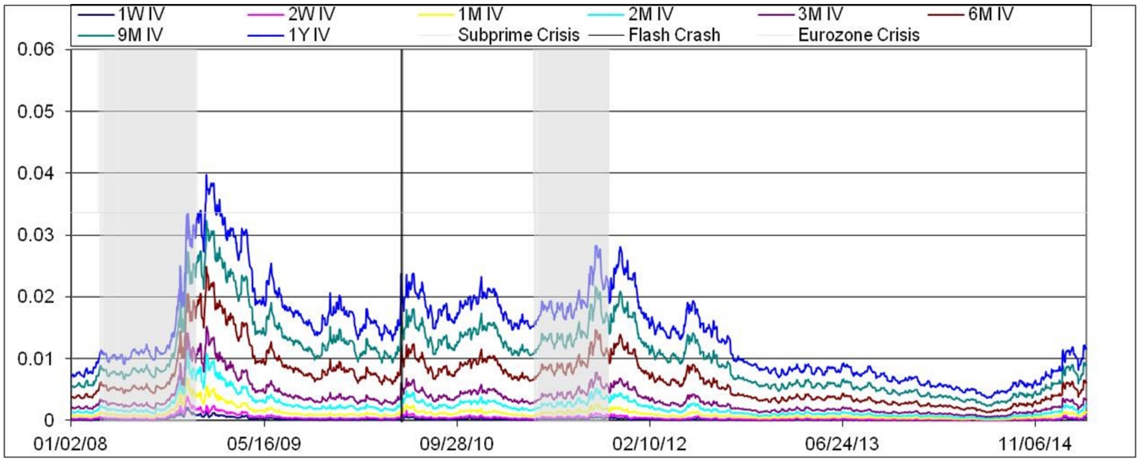


(a) M4 v.s. USD Influence Index

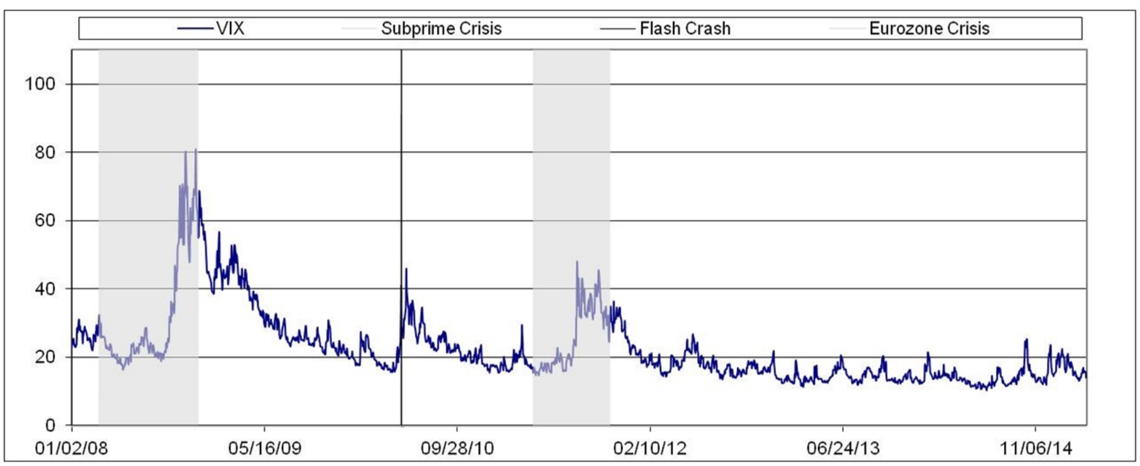


(b) M4 v.s. Change of USD Influence Index

Figure 5: US Dollar Influence Index and M4



(a) Black-Scholes IV



(b) VIX

Figure 6: BS IV v.s. VIX

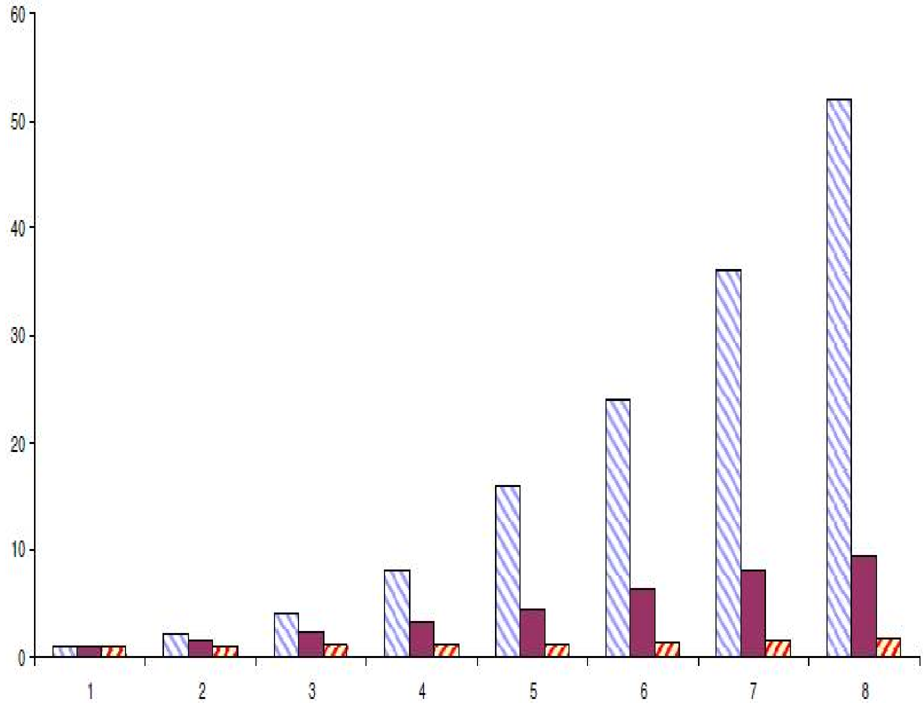
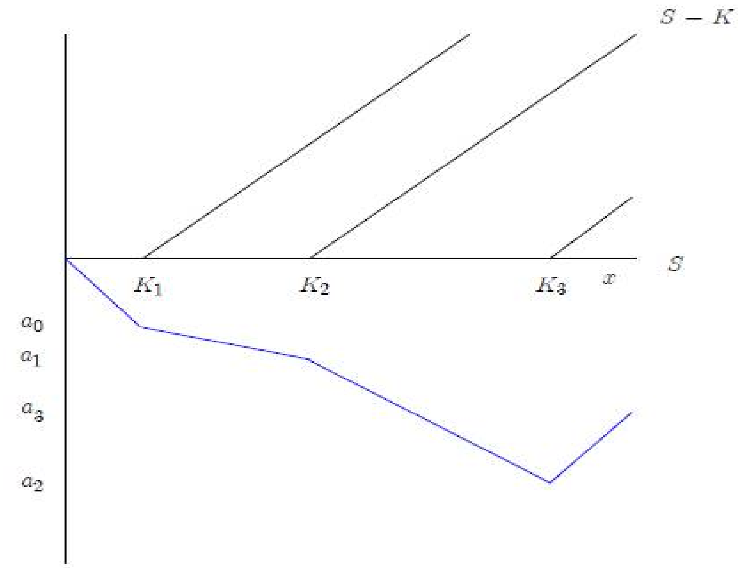
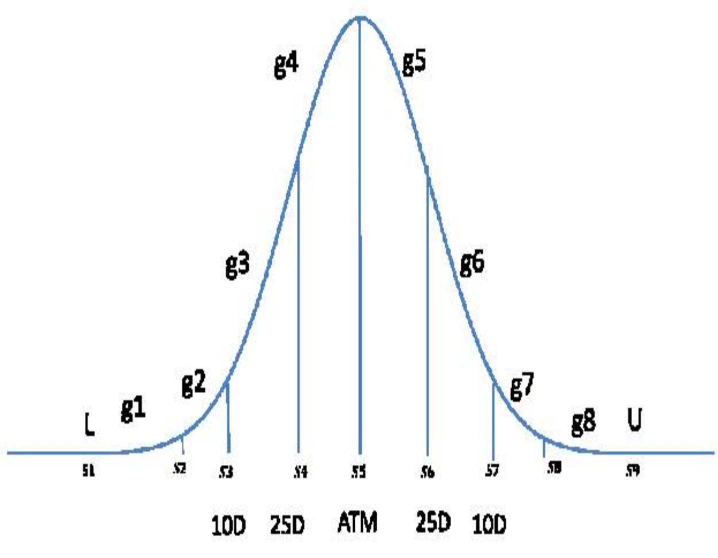


Figure 7: Variance Proportionality



(a) Piece-wise Linear Function



(b) Cubic Spine Function

Figure 8: Piece-wise Linear and Cubic Spline RNDs